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Properties of branes in curved spacetimes

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Abstract: A generic property of curved manifolds is the existence of focal points. We show that branes located at focal points of the geometry satisfy special properties. Examples of backgrounds to which our discussion applies are $AdS_m \times S^n$ and plane wave backgrounds. As an example, we show that a pair of AdS_2 branes located at the north and south pole of the S^5 in $AdS_5 \times S^5$ are half supersymmetric and that they are dual to a two-monopole solution of $\mathcal{N} = 4$ SU(N) SYM theory. Our second example involves spacelike branes in the (lorentzian) plane wave. We develop a modified lightcone gauge for the open string channel, analyze in detail the cylinder diagram and establish open-closed duality. In the new gauge the open string feels an inverted harmonic oscillator potential. When the branes are located at focal points of the geometry the amplitude acquires most of the characteristics of flat space amplitudes. In the open string channel the special properties are due to stringy modes that become massless.

Keywords: D-branes, Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

Understanding branes on curved backgrounds is an important problem. One of the most basic properties that one would like to understand is the interactions between branes and of branes with closed strings. In string perturbation theory the leading interaction is given by the cylinder diagram. Studying this diagram and its properties is important for a variety of reasons. Firstly, it can be viewed as either a one-loop open string diagram or as tree level exchange of closed strings. Showing that the two descriptions yield the same answer provides a non-trivial consistency check, and furthermore, this equivalence is the prototype example of a gauge theory/gravity duality. Secondly, possible divergences in the amplitude signal physical effects. For instance, in some cases cancelling such one-loop divergences leads to string-loop corrections to the beta function equations (and consequently corrections to the background) [1]–[3]. In other cases the divergence is a signal of an instability: a finite amplitude is obtained via analytic continuation but it has an imaginary part. The latter yields the decay rate of the brane.

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In flat spacetime the cylinder amplitude between two parallel branes exhibits the following behavior: (i) it is proportional to the volume of the D-branes; (ii) it vanishes for any separation if the branes are supersymmetric; (iii) the supersymmetries that annihilate the corresponding boundary state commute with time evolution. The first property is due to translational invariance of the interactions. Because of this factor the amplitude for infinitely extended branes diverges, but this divergence does not signify an instability or the onset of string-loop corrections. The second property is due to fermionic zero modes. The last property is dictated by the flat space superalgebra.

In curved spacetime none of these properties are expected to hold in general. For instance, even if individual branes are translationally invariant along their worldvolume, the system of a pair of branes will not in general retain this property. As a result the cylinder amplitude will not automatically be proportional to the worldvolume of the branes, and it becomes a non-trivial task to disentangle divergences that are due to the infinite volume of the brane from the physical divergences. One purpose of this work is analyze general properties of branes in curved spacetimes that would allow the better understanding of the meaning of the amplitudes.

In a general curved manifold we expect that special features appear when the branes are located at focal points of the background geometry. In these cases the system acquires new continuous zero modes: these are associated with open strings with ends on the two D-branes that lie along geodesics. Since any geodesic leaving from the original brane ends up on the other branes, there are new zero modes, namely the modes that parametrize the different geodesics. For the Neumann directions these are the position and velocity of the string ends, and for Dirichlet directions the position and velocity of $\partial_\sigma X^{r'}$ at the end of the string. In particular, the amplitudes will again be proportional to the volume of the branes since we have regained translational invariance along the worldvolume directions. Furthermore, if the branes are target space supersymmetric, supersymmetry will imply new fermionic zero modes, namely the fermionic partners for the new bosonic zero modes. Because of these modes, the cylinder amplitude is expected to vanish and we thus find that these amplitudes behave as in flat spacetime.

Focal points are a generic feature of curved manifolds. Examples include many of the backgrounds that enter in gravity/gauge theory dualities. Perhaps the prototype examples of curved manifolds with focal points are spheres: all geodesics that leave the north pole reconverge at the south pole. AdS spacetime itself has spacelike focusing points: timelike geodesics reconverge after global time π. Thus our discussion is relevant for branes in all backgrounds that involve AdS and/or spheres. In many of these cases we do not yet know how to solve string theory, so computing the cylinder diagram is out of reach. The previous discussion however implies that there are supersymmetric configurations that involve branes located at focal points of the geometry. This in turn implies via the gravity/gauge theory duality that there must exist dual supersymmetric configurations on the gauge theory side. We thus obtain an additional set of configurations that one should match between the two sides of the duality.

To illustrate this discussion we analyze AdS_2 branes on $AdS_5 \times S^5$. We have previously shown [4] that AdS_2 branes wrapping the time and radial coordinate of AdS_5 preserve 16 supercharges. Here we observe that a system of two such branes, one at the north pole and another at the south pole preserve the same number of supercharges. We then show that this configuration corresponds to a two monopole solution of the $\mathcal{N} = 4$ SU(N) SYM theory, which is also a half supersymmetric configuration.

Another set of examples of curved manifolds with focal points are plane waves. A particularly interesting case is the maximally supersymmetric background of IIB supergravity as it is the Penrose limit of $AdS_5 \times S^5$. The focal points of the geometry descend from corresponding focal points on $AdS_5 \times S^5$: geodesics that reconverge on *both* AdS_5 and the circle of S^5 along which we boost are part of the resulting plane wave spacetime. We thus expect that a pair of spacelike branes in the (lorentzian) plane wave exhibits special properties when located at the focal points.

This example has the advantage that string theory on this background is solvable in lightcone gauge and thus the cylinder diagram can be computed. In fact the computation of the cylinder diagram in the closed string channel for the branes of interest here was carried out in [5] and (as expected) none of the above mentioned flat space properties hold. Moreover, as we discuss in detail, the corresponding amplitudes are generically not just non-zero, they are infinite. Understanding the meaning of these infinities was one of the motivations of this work. The amplitude, however, recovers the flat space characteristics when the branes are located at focal points. In these cases, the infinities of the amplitudes are just due to the infinite volume of the branes. We expect that the infinities at generic separations are also of the same nature, even though they cannot be directly expressed as volume divergences.

Open strings in the standard lightcone gauge can only describe timelike branes that wrap both lightcone directions. In order to be able to analyze the open-closed duality we are led to develop a modified lightcone gauge where $X^+ \sim \sigma$. Recall that the worldsheet theory for closed strings in the standard lightcone gauge describes bosons and fermions in a harmonic oscillator potential. The open string theory however in the modified lightcone gauge describes open strings in an *inverted* harmonic oscillator potential. Throughout our analysis the worldsheet theory is lorentzian.

The special separations that on the closed string side correspond to focal points are mapped under open-closed duality to a specific value of the mass for the open string. As we approach this value of the mass one of the stringy modes becomes massless, and the special properties that were due to the infinite number of geodesics in the closed string channel are now due to the presence of extra massless modes.

On a generic background, the cylinder diagram between two supersymmetric branes which are related by symmetry transformations is not necessarily zero. This can even be the case for branes which are parallel, i.e. separated using a translational symmetry, if such translations do not commute with the supersymmetries.

Again these properties are nicely illustrated by the example of spacelike branes in the plane wave. In this case the relevant translational symmetry is the lightcone hamiltonian H which does not commute with target space supersymmetry. So if the boundary state $|B\rangle_0$ (defined at $x^+ = 0$) is annihilated by a combination of supercharges, the time-evolved state $|B, x^+\rangle = e^{-iHx^+}|B\rangle_0$, will generically be annihilated by a different set of supercharges.

As a result ${}_0\langle B|e^{-iHx^+}|B\rangle_0$ will in general be non-zero. However, exactly when the time of evolution is such that the geodesic focal point is reached, the set of supercharges that annihilates the boundary state rotates back to the original set. As a result the cylinder amplitude for two branes at this separation vanishes.

The physical relevance of the spacelike branes discussed here is unclear, since they exhibit known problematic features of spacelike branes, for instance, they source imaginary fluxes. Actually, as we discuss, these branes can be considered as E-branes of IIB* theory. Another (possibly related) problem is that the space of states of the corresponding open string contains negative norm states. Although the discussions here may clarify some of these features of spacelike branes, our main focus in this paper is on the generic properties of branes in curved backgrounds which the spacelike branes in the plane wave illustrate. We thus view these branes as a useful toy example, regardless of their physical significance, where string computations that illustrate the features of interest are possible.

This paper is organized as follows. In the next section 2 we discuss the example of the supersymmetric $AdS_2 - AdS_2$ configurations and their dual interpretation as a two monopole solution of $\mathcal{N} = 4$ SU(N) SYM theory. Then in section 3 we discuss spacelike branes in the plane wave. In particular, we develop the modified lightcone gauge in section 3.1. In section 3.2 we show that the spacelike branes discussed are actually E-branes of IIB* theory. In the remaining sections we analyze in detail the open-closed duality, the behavior of integrated amplitudes and the special properties when the branes are at distinguished separations.

2. Branes in AdS

A notable example of the phenomena discussed above is branes in $AdS \times S$ backgrounds which are separated on the sphere. The arguments of the previous section imply that a pair of branes located at antipodal points of the sphere have special properties. In particular if they preserve compatible supersymmetries the system should be supersymmetric and stable. This leads to a number of new supersymmetric brane configurations which we will illustrate via the specific case of AdS_2 branes in an $AdS_5 \times S^5$ background, although we will also discuss generalizations at the end of this section. One should note that since these particular branes decouple in the Penrose limit [4] this case is not related to branes in the plane wave.

2.1 A supersymmetric AdS_2-AdS_2 configuration

It was shown in [4] that a given AdS_2 brane located at any point in the S^5 preserves half of the supersymmetries. Now consider two such branes separated on the S^5. There is clearly a distinguished configuration in which the branes are at antipodal points on the sphere. For generic separations there is precisely one geodesic between the branes, whilst for this configuration there is an infinite family of geodesics since the second brane is placed at a focusing point. This behavior should be reflected in the spectrum of open strings stretching between the branes: at the antipodal separations one should get a family of zero modes for the spherical coordinates whilst for generic separations there are no Dirichlet zero modes.

Antipodal separations are also distinguished by supersymmetry. Let us write the $AdS_5 \times S^5$ metric as

$$ds^2 = R^2 \left(\frac{du^2}{u^2} + u^2 (dx \cdot dx)_4 + d\theta_1^2 + \sum_{k=2}^{5} \prod_{j=1}^{k-1} \sin\theta_j^2 d\theta_k^2 \right). \quad (2.1)$$

Following the conventions of [4], the Killing spinors can be written as

$$\epsilon = -u^{-1/2} \Gamma_4 h(\theta_a) \lambda_2 + u^{1/2} h(\theta_a)(\lambda_1 + x \cdot \Gamma_x \lambda_2), \quad (2.2)$$

where Γ_m are tangent space gamma matrices (4 is the radial direction) and (λ_1, λ_2) are constant complex spinors of negative and positive chirality respectively such that

$$\lambda_1 = \lambda_1^R - i\Gamma^{0123}\lambda_1^R; \qquad \lambda_2 = \lambda_2^R + i\Gamma^{0123}\lambda_2^R, \quad (2.3)$$

for real $(\lambda_1^R, \lambda_2^R)$. The function $h(\theta_a)$ is given by

$$h(\theta_a) = \exp\left(\frac{1}{2}\theta_1 \Gamma^{45}\right) \exp\left(\frac{1}{2}\theta_2 \Gamma^{56}\right) \exp\left(\frac{1}{2}\theta_3 \Gamma^{67}\right) \exp\left(\frac{1}{2}\theta_4 \Gamma^{78}\right) \exp\left(\frac{1}{2}\theta_5 \Gamma^{89}\right). \quad (2.4)$$

The AdS_2 branes we discuss extend along the time and radial direction of AdS_5 and they are located at a constant position in transverse space, $(x_0^i, \theta_a), i{=}1,2,3,\ a{=}1,\ldots,5$. The supersymmetries preserved by an AdS_2 brane can be determined via the kappa symmetry projection to satisfy

$$\lambda_1^R = \eta e^{-\theta_1 \Gamma^{45}} \Gamma^{1234} \lambda_1^R; \qquad \lambda_2^R = \eta e^{-\theta_1 \Gamma^{45}} \Gamma^{1234} \lambda_2^R - 2\eta e^{-\theta_1 \Gamma^{45}} x_0^i \Gamma^i \lambda_1^R, \quad (2.5)$$

where $\eta = \pm 1$ for a brane/anti-brane, respectively.[1]

So as one moves a brane from the north pole of the sphere the supersymmetries preserved by it are rotated, until at the south pole it preserves precisely the opposite supersymmetries: it has become an anti-brane. Therefore, a brane (i.e. $\eta = 1$) located at $\theta_1 = 0$ and an anti-brane ($\eta = -1$) at $\theta_1 = \pi$, located at the same transverse positions x_0^i preserve exactly the same sixteen supersymmetries. This should be reflected in the presence of fermion zero modes in the open string spectrum for this configuration, and the vanishing of exchange diagrams between these branes.

Furthermore, if these branes are separated in their transverse positions they still preserve eight supercharges. Branes at the same x_0^i position preserve eight of the ordinary supercharges and eight of the conformal supercharges, reflecting the residual unbroken conformal invariance $\mathrm{SO}(2,1) \subset \mathrm{SO}(2,4)$. Separated AdS_2 branes however break the remaining conformal invariance and conformal supersymmetries.

[1] Note that the worldvolume theory of a brane is distinguished from that of an anti-brane by the sign of the Wess-Zumino term. Each action is invariant under kappa symmetry but the transformations differ as $\delta\theta = (1 - \eta\Gamma)\kappa$.

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2.2 Dual description as a two-monopole configuration

The dual description of the AdS_2 brane is as a monopole in the gauge theory [6, 7, 4], and the properties described above correspond to known properties of monopoles in $\mathcal{N} = 4$ SYM theory. The relevant fields are the vector A_μ and the six scalars X^A which transform in the **6** of the SO(6) R symmetry. One can define an SO(6) vector of magnetic charges

$$T^A = \frac{1}{v} \int_\Sigma d\Sigma^{ij} \, \mathrm{Tr}\, F_{ij} X^A \,, \tag{2.6}$$

where the integral is over a closed spatial surface that encloses the monopole and $v^2 = \langle X^2 \rangle$ is the magnitude of the vev of the scalars at infinity.

Now recall the asymptotic behavior as $r \to \infty$ of an elementary single monopole in gauge group SU(2) located at the origin in R^3:

$$F_{ij}^a \sim \frac{g}{r^4} \epsilon_{ijk} x^a x^k \,; \qquad X^{aA} \sim \eta \nu^A v \frac{x^a}{r} \,, \tag{2.7}$$

where g is the magnetic change, a is the gauge group index and x^i parametrize the spatial R^3 with $x^i x^i = r^2$. The unit vector ν^A describes the SO(6) direction in which the monopole is pointing whilst $\eta = \pm 1$ with the two signs corresponding to monopole and anti-monopole, respectively. Fixing convenient normalizations the mass of a single monopole m is given by $m = vg$. More generally the BPS bound may be stated as [8]

$$M^2 = v^2 \left(T^A T^A + Q^A Q^A \right) , \tag{2.8}$$

where Q^A denote the electric charges (defined as in (2.6) but with $F \to *F$).

Let us discuss more generally monopoles in SU(N) gauge theory. The gauge group is considered to be Higgsed to the maximal torus $U(1)^N$. Recall that the monopole charges are represented by a set of $N-1$ integers, $(m_1, \ldots, m_{N-1})$. One can obtain monopole solutions by embedding SU(2) solutions in the SU(N) theory. The details of the construction [9] are not needed here, but we briefly review the results. Each simple root of the $su(N)$ defines an SU(2) subgroup and the corresponding monopole solution carries a unit of magnetic charge. These are the fundamental monopole solutions. All other roots are associated with a superposition of fundamental monopole solutions at the same point. Writing the root as a sum of simple roots one obtains the constituency of the multi-monopole solution. $su(N)$ has $N-1$ simple roots α^a and $(N-2)(N-1)/2$ positive (non-simple) roots given by $\sum_{a=b}^c \alpha^a$, $b < c$. Any of these roots is associated with a superposition of $(c - b + 1)$ elementary monopoles. In particular, there are $(N-2)$ two monopole solutions. Explicit expressions for solutions are given in [9], but these will not be needed here.

So far we did not consider the effect of the global SO(6) symmetry. Consider the solution corresponding to the superposition of one monopole and one anti-monopole pointing in a different direction in the SO(6). Vector addition of the magnetic charges implies that

$$|T^A| = 2g \sin\left(\frac{\theta}{2} + \frac{\pi}{4}(1 + \eta) \right) , \tag{2.9}$$

where θ is the angle between the two directions. Since both the monopole and the anti-monopole have mass equal to g, the BPS bound above is clearly saturated if the second object is an anti-monopole and the angular separation is $\theta = \pi$, and in this case the total charge is two. The other possibility is if the second object is a monopole at the same angular position.

These facts have a very simple explanation in terms of D-branes [6]. Let us consider a configuration of N D3 branes. The worldvolume theory is a $\mathcal{N} = 4$ SU(N) theory. We now Higgs the theory by separating the branes in one direction, say X^1,

$$X^{1a} = \mu_a \,, \qquad a = 1, \dots, N \,. \tag{2.10}$$

As discussed in [6], the elementary monopole solutions correspond to D1-branes stretched between consecutive D3 branes $(a, a+1)$. D1-branes stretched between two non-consecutive D3-branes (a, b) correspond to a superposition of $b - a + 1$ elementary monopoles and carry magnetic charge $(\vec{0}_{a-1}, \vec{1}_{b-a+1}, \vec{0}_{n-b-1})$, where $\vec{A}_a$ denotes a row vector with a entries equal to A. The D1 brane can be considered as a collection of D1 branes $(a, a+1), \dots, (b-1, b)$ with endpoints identified. Notice that a brane ending on a pair of D-branes induces a monopole on one and an anti-monopole in the other. The direction of the monopoles, however, in the SO(6) space is also opposite and as we discussed in the previous paragraph the total magnetic charge adds up.

Let us now consider the configuration of $N - 2$ coincident branes and two branes separated in opposite directions such that $X^2 = v^2$ and take the near-horizon limit. The $N-2$ coincident branes are replaced by $AdS_5 \times S^5$, the two separated branes were pushed to infinity and the D1 branes become the two AdS_2 branes located at the antipodal points of the S^5 (recall that the position on the sphere of the AdS_2 brane is mapped to the direction of the scalar field of the gauge theory monopole). Thus we find direct agreement with the bulk result.

When the monopoles are separated in the R^3, the scale introduced necessarily leads to the breaking of conformal symmetry and of the conformal supersymmetries. Nonetheless the configuration is 1/4 BPS.

Under S duality the AdS_2 D-brane becomes a (p, q) AdS_2 string, whilst the monopole is mapped into a (p, q) dyon in the gauge theory. These objects should exhibit similar properties, which could be demonstrated for the strings using the manifestly $SL(2, Z)$ covariant formulation of [10].

2.3 Generalizations

The discussion here generalizes to all cases where the background involves spheres. A particularly interesting class of such examples are branes on $AdS_k \times S^l \times S^m \times T^n$ for $k, l, m = 2, 3$ (with one or two sphere factors and n such that the spacetime has $D = 10$). These spacetimes are derived as a near-horizon limit of brane intersections and are exact solutions of string theory, i.e. there are WZW models associated with them [11]. Branes on S^3 and AdS_3 have been extensively analyzed in recent years, see [12] for an (incomplete) set of references. In these cases one should be able to go beyond the supergravity approximation and explicitly compute the relevant string amplitudes. We will not pursue this here. Instead we will discuss a different set of examples where the exact computation of cylinder is also possible, namely branes in the plane wave background of IIB supergravity.

Another generalization involves spacelike branes on AdS spacetimes. Recall that (the universal cover of) global AdS_{d+1} has the metric

$$ds^2 = R^2 \left(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_{d-1}^2\right), \qquad 0 \le \rho < \infty,\ -\infty < t < +\infty . \quad (2.11)$$

Timelike geodesics are periodic with period 2π and they reconverge after $t = \pi$ [13]. (The former follows from the fact that before we take the universal cover the time coordinate had range $[-\pi, \pi]$). It follows that the system of spacelike branes located at $\rho = 0, t = k\pi,\ k \in \mathbb{Z}$ should exhibit special properties. In the next few sections we analyze related branes in the plane wave background of IIB supergravity.

3. Spacelike branes in the plane wave

The second example that we analyze in detail is the case of spacelike branes in the maximally supersymmetric plane wave background. For our purposes it is crucial that the branes are spacelike and are in a lorentzian background. The reason is that (as we discuss in detail later) the lorentzian spacetime has focal points in the x^+ direction. Wick rotating to an euclidean section by $x^+ \to ix^+$, so that one would now be discussing D-instantons, results in geodesics that are no longer periodic in x^+.

The boundary states for the branes under consideration have been discussed previously in [14, 5, 15]. Here the emphasis is on the fact that the worldsheet theory and the target spacetime are lorentzian. The standard lightcone gauge does not allow for a description of spacelike branes in the open string channel. We thus develop in detail in the next subsection a modified lightcone gauge that allows for such a description.

The branes we discuss have imaginary couplings to the RR fields. This can be read off from the corresponding boundary state. So these branes are not S-branes. They are however E-branes: they have real couplings when considered as branes of the IIB* plane wave. In fact we show in section 3.2 that (formal) T-dualities along the lightcone map the lorentzian $(+,-,m,n)$ branes of the IIB plane wave to the euclidean (m,n) branes of the IIB* plane wave.

3.1 Open strings in a modified lightcone gauge

In this section we discuss a modified bosonic lightcone gauge, appropriate for describing certain classes of D-branes and for checking open/closed duality. We will discuss the use of this gauge for strings in the plane wave; the flat space case is also clearly contained in this discussion by setting the mass parameters to zero. The action for strings in the plane wave, with Brinkmann metric

$$ds^2 = 2dx^+dx^- + \sum_{I=1}^{8} \left(-\mu^2 \left(x^I\right)^2 \left(dx^+\right)^2 + \left(dx^I\right)^2\right), \quad (3.1)$$

and RR flux

$$F_{+1234} = F_{+5678} = 4\mu\,, \tag{3.2}$$

in fermionic lightcone gauge is [17]

$$\begin{aligned} S = T\int_\Sigma d^2\sigma\Big(&-\frac{1}{2}\sqrt{-g}g^{ab}\left(2\partial_a x^+\partial_b x^- - \mu^2 x_I^2\partial_a x^+\partial_b x^+ + \partial_a x^I\partial_b x^I\right) - \\ &- i\sqrt{-g}g^{ab}\partial_b x^+(\bar\theta\partial_a\theta + \theta\partial_a\bar\theta + 2i\mu\partial_a x^+\bar\theta\Pi\theta) + i\epsilon^{ab}\partial_a x^+(\theta\partial_b\theta + \bar\theta\partial_b\bar\theta)\Big)\,. \end{aligned} \tag{3.3}$$

In this expression g_{ab} is the worldsheet metric with (τ,σ) the worldsheet coordinates and $\epsilon^{01}=1$. (x^+, x^-, x^I) are the bosonic coordinates of the target superspace. $\theta = \frac{1}{\sqrt{2}}(\theta^1 + i\theta^2)$ is a complex SO(8) spinor of positive chirality. The 8×8 matrices $\gamma^I_{a\dot b}$ and its transpose $\tilde\gamma^I_{\dot a b}$ are the off-diagonal components of the 16×16 SO(8) γ matrices and couple SO(8) spinors of opposite chirality. The matrix $\Pi = \gamma^1\tilde\gamma^2\gamma^3\tilde\gamma^4$. Fixing $\alpha' = 1$, T is the inverse length of the string, which we choose to be 2π (π) for a closed (open) string respectively.

The standard lightcone gauge choice $x^+ = p^+\tau$ with conformal gauge $g_{ab} = \eta_{ab}$ leads to an action for free massive fields. This is however not the only simplifying gauge choice: the more general gauge choice

$$x^+ = x_0^+ + p^+\tau + r^+\sigma\,, \tag{3.4}$$

along with the conformal gauge $g_{ab} = \eta_{ab}$ also leads to a purely quadratic action. With such a gauge choice the action becomes

$$\begin{aligned} S = T\int d^2\sigma(&p^+\partial_\tau x^- - r^+\partial_\sigma x^- + \frac{1}{2}\left(\left(\partial_\tau x^I\right)^2 - \left(\partial_\sigma x^I\right)^2 - \mu^2 a_+ a_-(x^I)^2\right) + \\ &+ i\left(a_-\theta^1\partial_+\theta^1 + a_+\theta^2\partial_-\theta^2 - 2\mu a_- a_+\theta^1\Pi\theta^2\right)\,, \end{aligned} \tag{3.5}$$

where $a_\pm = (p^+ \pm r^+)$. So far we have not imposed any worldsheet periodicity or boundary conditions. For closed strings the gauge choice will only be consistent with periodicity when $r^+ = 0$ (unless the spacetime coordinate x^+ is compactified). Thus in the closed string channel one should use the standard lightcone gauge choice, which immediately enforces that any boundary states at fixed τ are Dirichlet in x^+.

In the open string channel, the general gauge choice can be applied. However, one is usually interested in describing D-branes with pure Neumann or pure Dirichlet boundary conditions at fixed σ. For these cases, one must impose the gauge choices $r^+ = 0$ and $p^+ = 0$ respectively.

An interesting alternative possibility would be to impose $a_+ = 0$ or $a_- = 0$. We will not explore this here, except for the following comments. These cases correspond to a "hybrid" gauge where the static gauge $X^0 = \tau$, $X^1 = \sigma$ (for $a_+ = 0$, when $a_- = 0$ we have $X^1 = -\sigma$) is chosen for the bosons and fermionic lightcone gauge is chosen for the fermions. This gauge however appears somewhat singular as half of the fermions drop out completely from the action and it may not be an admissible gauge, see a related discussion in [18] where such a hybrid gauge for M2 branes is considered. Furthermore, the Virasoro constraints are more complicated than in the standard lightcone gauge.

Recall that the action in standard lightcone gauge $r^+ = 0$ is given by

$$S[p^+] = T\int d^2\sigma\Big(p^+\partial_\tau x^- + \frac{1}{2}((\partial_\tau x^I)^2 - (\partial_\sigma x^I)^2 - m^2(x^I)^2) + \\ + ip^+(\theta^1\partial_+\theta^1 + \theta^2\partial_-\theta^2 - 2m\theta^1\Pi\theta^2)\Big)\,, \tag{3.6}$$

where $m = \mu p^+$. The resulting action in the new gauge $p^+ = 0$ is

$$S[r^+] = T\int d^2\sigma\Big(-r^+\partial_\sigma x^- + \frac{1}{2}((\partial_\tau x^I)^2 - (\partial_\sigma x^I)^2 + \tilde{m}^2(x^I)^2) - \\ - ir^+(\theta^1\partial_+\theta^1 - \theta^2\partial_-\theta^2 - 2\tilde{m}\theta^1\Pi\theta^2)\Big). \tag{3.7}$$

Here $\partial_\pm = \partial_\tau \pm \partial_\sigma$ and $\tilde{m} = \mu r^+$. Notice that the standard light cone gauge leads to a system of 8 free bosons and 8 free fermions in a harmonic oscillator potential, whilst the new lightcone action describes the same degrees of freedom but in an *inverted* harmonic oscillator potential. The two actions are formally related by $m \to i\tilde{m}$ and $\theta^2 \to -i\theta^2$. They also differ in x^- boundary terms; the latter are negligible in (3.6) since they are total time derivatives but not in (3.7) where they are spatial derivatives.

The field equations from (3.7) are

$$\begin{aligned}(\partial_\tau^2 - \partial_\sigma^2)x^I &= \tilde{m}^2 x^I\,;\\ \partial_+\theta^1 &= \tilde{m}\Pi\theta^2\,; \qquad \partial_-\theta^2 = \tilde{m}\Pi\theta^1\,,\end{aligned} \tag{3.8}$$

whilst one can show that the gauge fixed Virasoro constraints are

$$T_{\sigma\tau} = r^+[\partial_\tau x^- + i(\theta^1\partial_\tau\theta^1 + \theta^2\partial_\tau\theta^2)] + \partial_\tau x^I\partial_\sigma x^I = 0\,; \tag{3.9}$$

$$T_{\tau\tau} = \mathcal{H}_o = \left[r^+\partial_\sigma x^- + ir^+\left(\theta^1\partial_\sigma\theta^1 + \theta^2\partial_\sigma\theta^2 - 2\tilde{m}\theta^1\Pi\theta^2\right)\right] + \\ + \frac{1}{2}((\partial_\tau x^I)^2 + (\partial_\sigma x^I)^2 - \tilde{m}^2(x^I)^2) = 0\,. \tag{3.10}$$

Note that the latter is the canonical hamiltonian for the action (3.7). It is convenient to write

$$\hat{\mathcal{H}}_o = \frac{1}{2}\left(\left(\partial_\tau x^I\right)^2 + \left(\partial_\sigma x^I\right)^2 - \tilde{m}^2(x^I)^2\right) + ir^+\left(\theta^2\partial_\tau\theta^2 - \theta^1\partial_\tau\theta^1\right), \tag{3.11}$$

so that $\mathcal{H}_o = r^+\partial_\sigma x^- + \hat{\mathcal{H}}_o = 0$ when one imposes the fermion field equations.

Consistent boundary conditions at $\sigma = 0, \pi$ following from the variational problem are pure Neumann ($\partial_\sigma X^r = 0$) and Dirichlet ($\partial_\tau X^{r'} = 0$) conditions for the bosons and $\theta^1 = iM\theta^2$ for the fermions, where M is an orthogonal matrix which is the product of gamma matrices $\gamma^{r'}$ where r' are the Dirichlet directions transverse to the lightcone. (3.9) then immediately enforces that x^- is Dirichlet, as x^+ manifestly also is. The physical meaning of these boundary conditions is clear: we are quantizing open strings stretched between two *spacelike* branes in lorentzian spacetime, separated along the lightcone as well as in spatial directions. Note that our convention is as in [19] that an Ep-brane has p longitudinal spacelike directions.

As discussed in [4], branes in the plane wave are naturally divided into classes depending on which directions transverse to the lightcone they span. Here we focus on $(m, m+2)$ branes for which $(M\Pi)^2 = -1$; in the classification of [15] and [20]–[23] these are D_- branes. Since we are interested in generic properties of spacelike branes which depend on the background geometry rather than the specific class of branes we consider, it is convenient to focus on one class of branes.

For generic r^+ the mode expansions are then as follows. For the bosons,

$$x^r(\sigma,\tau) = x_0^r \cosh(\tilde{m}\tau) + \tilde{m}^{-1} p_0^r \sinh(\tilde{m}\tau) + i \sum_{n\neq 0} \omega_n^{-1} \alpha_n^r e^{-i\omega_n \tau} \cos(n\sigma)\,; \tag{3.12}$$

$$\begin{aligned} x^{r'}(\sigma,\tau) &= x_1^{r'} \cos(\tilde{m}\sigma) + \left(x_2^{r'} \operatorname{cosec}(\tilde{m}\pi) - x_1^{r'} \cot(\tilde{m}\pi) \right) \sin(\tilde{m}\sigma) + \\ &\quad + \sum_{n\neq 0} \omega_n^{-1} \alpha_n^{r'} e^{-i\omega_n \tau} \sin(n\sigma). \end{aligned} \tag{3.13}$$

In these expressions, $\omega_n = \operatorname{sgn}(n)\sqrt{n^2 - \tilde{m}^2}$. When $\tilde{m}^2 < 1$, i.e. when the mass scale set by the flux is smaller than the string mass, all stringy modes have real frequencies. In this regime, the upside down harmonic oscillator potential affects mostly the zero modes: the zero modes exhibit an exponential behavior but all stringy modes are oscillatory, as in flat space. When $\tilde{m}^2 > 1$ stringy modes with $n^2 < \tilde{m}^2$ exhibit exponential behavior. This case can be included in the generic analysis with $\omega_n \to i\operatorname{sgn}(n)\sqrt{\tilde{m}^2 - n^2}$. The generic analysis breaks down when $\tilde{m}$ is integral. In this case one stringy mode becomes massless. We will discuss in detail the physical significance of these values later. For concreteness, we consider $\tilde{m}^2 < 1$ in the generic analysis.

The commutation relations are

$$[x_0^r, p_0^s] = i\delta^{rs}\,; \qquad [\bar{a}_0^r, a_0^s] = \delta^{rs}\,; \qquad [a_n^I, a_l^J] = \operatorname{sgn}(n)\delta_{n+l}\delta^{IJ}\,, \tag{3.14}$$

where one defines (as noted, $\tilde{m}$ is assumed non-integral, so $\omega_n \neq 0$).

$$a_0^r = e^{-i\pi/4}\frac{1}{\sqrt{2\tilde{m}}}(p_0^r - \tilde{m}x_0^r)\,, \qquad \bar{a}_0^r = e^{-i\pi/4}\frac{1}{\sqrt{2\tilde{m}}}(p_0^r + \tilde{m}x_0^r)\,, \qquad a_n^I = \sqrt{\frac{1}{|\omega_n|}}\alpha_n^I\,. \tag{3.15}$$

The phases in the definition of a_0 and $\bar{a}_0$ are needed in order for their commutator to be real. The fermion mode expansions are the following

$$\begin{aligned} \sqrt{r^+}\theta^1 &= \theta_0 \cosh(\tilde{m}\tau) + \tilde{\theta}_0 \sinh(\tilde{m}\tau) + \sum_{n\neq 0} c_n^2 \left(d_n(d_n - M\Pi)\theta_n\phi_n + \tilde{\theta}_n\tilde{\phi}_n \right); \\ \sqrt{r^+}\theta^2 &= \Pi\tilde{\theta}_0 \cosh(\tilde{m}\tau) + \Pi\theta_0 \sinh(\tilde{m}\tau) + \sum_{n\neq 0} c_n^2 \left(-id_n\Pi\tilde{\theta}_n\tilde{\phi}_n + i(M^t + d_n\Pi)\theta_n\phi_n \right), \end{aligned} \tag{3.16}$$

where

$$\begin{aligned} \phi_n &= e^{-i(\omega_n\tau + n\sigma)}\,, & \tilde{\phi}_n &= e^{-i(\omega_n\tau - n\sigma)}\,; \\ d_n &= \frac{1}{\tilde{m}}(\omega_n - n)\,, & c_n &= \frac{1}{\sqrt{1+d_n^2}}. \end{aligned} \tag{3.17}$$

Imposing the boundary conditions one gets $\tilde{\theta_0} = iM\Pi\theta_0$ and $\tilde{\theta}_n = -(1 + d_n M\Pi)\theta_n$ whilst the anticommutators are given by

$$\left\{\theta_0^a, \theta_0^b\right\} = \frac{1}{4}\delta^{ab}; \qquad \left\{\theta_n^a, \theta_m^b\right\} = \frac{1}{4}\delta^{ab}\delta_{n+m}. \tag{3.18}$$

Because of the fermion boundary condition,[2] $\theta^1| = iM\theta^2|$, the fermions cannot be taken to be real. This reflects the fact that the brane is spacelike. However, one can still impose a modified reality condition:

$$(\theta_a^1)^* = A_a^b\theta_b^1, \qquad (\theta_a^2)^* = B_a^b\theta_b^2, \tag{3.19}$$

where A and B are 8×8 real matrices, i.e. the complex conjugate of the spinor is not the spinor itself but a real rotation of it. The matrices A and B should be equal to a sum of even powers of gamma matrices so that they do not change the chirality of the spinor. Compatibility with the field equations, boundary conditions and the fact that $*$ should be an involution yields (after some manipulations),

$$A^2 = 1, \qquad B = \Pi A\Pi, \qquad \{M\Pi, A\} = 0. \tag{3.20}$$

One can obtain the action of the new reality condition on the modes as

$$(\theta_0)^* = A\theta_0, \qquad (\theta_n)^* = A\theta_{-n}. \tag{3.21}$$

The bosonic oscillators satisfy the standard reality conditions, i.e. p_0^r and x_0^r are real and $a_n^* = a_{-n}$, but because of the phase factors in (3.15) a_0 and $\bar{a}_0$ satisfy unconventional reality conditions,

$$a_0^* = -ia_0, \qquad \bar{a}_0^* = -i\bar{a}_0. \tag{3.22}$$

The solution to (3.20) differs depending on whether $M^2 = 1$ or $M^2 = -1$. The former case corresponds to E4 branes and in this case

$$M^2 = 1: \qquad A = M, \quad B = -M. \tag{3.23}$$

Thus in this case the reality condition is consistent with the isometry group preserved by the E4 brane. The case $M^2 = -1$ is relevant for E2 and E6 branes. It is easy to see that (3.20) admits a solution in all cases, but one has to select two directions, one in each SO(4). Consider for example a $(2,0)$ brane extending along the 1 and 2 directions. Then one has to select two directions, one in each SO(4) and both transverse to the brane. For instance one may select the directions 4 and 8, so that $A = \gamma^{3567}$ (and consequently $B = -\gamma^{1248}$) is a solution. Thus in this case a choice of a reality condition breaks further the (bosonic) isometry group.

With an appropriate choice of basis, $A = \sigma^3 \otimes 1_4$, $M\Pi = i\sigma^2 \otimes 1_4$, where σ^i are Pauli matrices and 1_4 is a 4×4 matrix. Let us also define

$$\theta = \begin{pmatrix} \phi \\ i\lambda \end{pmatrix}, \tag{3.24}$$

[2] $\theta|$ indicates evaluation of θ at $\sigma = 0, \pi$.

where ϕ and λ are 4-component spinors. The reality condition (3.21) implies

$$\phi_n^* = \phi_{-n}\,, \qquad \lambda_n^* = \lambda_{-n}\,. \tag{3.25}$$

In terms of this decomposition the anticommutation relations read

$$\{\phi_0, \phi_0\} = \frac{1}{4}\,, \qquad \{\lambda_0, \lambda_0\} = -\frac{1}{4}\,, \qquad \{\phi_n, \phi_{-n}\} = \frac{1}{4}\,, \qquad \{\lambda_n, \lambda_{-n}\} = -\frac{1}{4}\,, \tag{3.26}$$

where we suppress the spinor indices. Thus the λ-modes have ghost-like anticommutation relations and the state space has indefinite metric.

The generator of σ-translations is given by the conserved charge $\hat{H}_o = \frac{1}{\pi}\int_0^\pi d\sigma \hat{\mathcal{H}}_o$ with mode expansion

$$\begin{aligned}
\hat{H}_o &= h_D + h_0 + h_N\,;\\
h_D &= \frac{\tilde{m}}{2\pi\sin(\tilde{m}\pi)}\sum_{r'}\left(\cos(\tilde{m}\pi)((x_1^{r'})^2 + (x_2^{r'})^2\right) - 2x_1^{r'}x_2^{r'})\,;\\
h_0 &= \frac{1}{2}\sum_{r=1}^{p}((p_0^r)^2 - \tilde{m}^2(x_0^r)^2) + 2\tilde{m}\theta_0 M\Pi\theta_0 = \frac{1}{2}\sum_{r=1}^{p}\left((p_0^r)^2 - \tilde{m}^2(x_0^r)^2\right) + 4i\tilde{m}\phi_0\lambda_0\,;\\
h_N &= \sum_{n>0}\left(\omega_n a_{-n}^I a_n^I + 4\omega_n\theta_n\theta_{-n}\right) = \sum_{n>0}\left(\omega_n a_n^{I\,\dagger} a_n^I + 4\omega_n\left(\phi_n\phi_n^\dagger - \lambda_n\lambda_n^\dagger\right)\right).
\end{aligned} \tag{3.27}$$

which is clearly hermitean but not positive definite.

In proceeding to quantize the system one is faced with the problem that the hamiltonian is unbounded from below and half of the fermionic modes satisfy ghostlike anticommutation relations. As we discuss in the next section these branes are T-dual along the lightcone directions to standard timelike branes. T-duality suggests that the appropriate quantization is the "analytic" continuation of the quantization of the string in the standard harmonic oscillator potential. One could argue that the problems we encounter here are associated with the fact that one of the T-dualities is timelike. Proceeding in this way we define the vacuum by

$$\bar{a}_0\,|0\rangle = a_n^I\,|0\rangle = 0\,, \qquad \theta_{-n}\,|0\rangle = 0 \qquad (1 - iM\Pi)\theta_0\,|0\rangle = \theta_0^-\,|0\rangle = 0\,, \tag{3.28}$$

where we define $\theta_0^\pm = 2(\phi_0 \pm \lambda_0)$. These modes satisfy the anticommutation relations,

$$\{\theta_0^+, \theta_0^-\} = 1\,, \qquad \{\theta_0^\pm, \theta_0^\pm\} = 0\,. \tag{3.29}$$

In summary, we consider $\bar{a}_0, \theta_0^-, a_n, \theta_{-n}$ as annihilation operators and $a_0, \theta_0^+, a_{-n}, \theta_n$ with negative n as creation operators. $|0\rangle$ is annihilated by all annihilation operators and $\langle 0|$ by all creation operators. We now build the Fock space by acting on $|0\rangle$ with the creation operators (or on $\langle 0|$ by annihilation operators). Notice that the bar and ket states are not related by conjugation (because of the fermion zero modes), but there is a natural inner product.

The spectrum constructed this way is identical to the spectrum of lorentzian $(+, -, m, n)$ [15] but the eigenvalues are related by $m \to i\tilde{m}$. In particular, the states generated by the zero modes have imaginary eigenvalues w.r.t. $\hat{H}_0$.[3] This is not in contradiction with the fact that $\hat{H}_0$ is formally hermitean, as the state space has indefinite metric.

3.2 T-duality and relation with E-branes of type IIB* theory

In flat space, one can also view the spacelike branes as being related by formal T-duality in the (x^+, x^-) directions to the usual lorentzian branes. Under such T-duality, in one timelike and one spacelike direction, the type IIB theory is mapped to the type IIB* theory [19] in which the RR fields have opposite sign kinetic terms to usual. Thus the boundary states describe E-branes in the type IIB* theory. Notice that this argument applies irrespectively of whether one is using the lightcone GS or the RNS description. A detailed comparison between the lightcone GS and RNS descriptions of branes satisfying Dirichlet conditions along the time direction for the case of $p = -1$ can be found in [24], and it seems likely that the conclusions of this paper extend to all other p. Provided that this is the case, some of the boundary states that have been proposed to describe S-branes, such as the RNS boundary state given in [25, section 4.1] which is pure Dirichlet in the time direction, contain imaginary couplings to the RR-fields and as such should be associated with E-branes and not S-branes.

In the plane wave the same interpretation holds. To see this we first work out how the T dualities act on the plane wave background. We define new coordinates $x^\pm = \frac{1}{\sqrt{2}}(\pm t + u)$ and T-dualize on (u, t) using standard T duality rules [26]. This (formal) procedure results in the following solution for the T dual background:

$$\begin{aligned} ds^2 &= 2d\tilde{x}^+ d\tilde{x}^- + \sum_{I=1}^{8} \left(\mu^2 (x^I)^2 (d\tilde{x}^+)^2 + (dx^I)^2 \right); \\ \hat{F}_{+1234} &= \hat{F}_{+5678} = 4\mu\,. \end{aligned} \tag{3.30}$$

The dual lightcone coordinates are related to the dual $(\tilde{t}, \tilde{u})$ coordinates as $\tilde{x}^\pm = \frac{1}{\sqrt{2}}(\mp\tilde{t} + \tilde{u})$. There is also an imaginary shift by $\pi/2$ of the dilaton, but this just reflects the fact that the NSNS and RR fields in the IIB* theory have opposite signs for their kinetic terms. The difference compared to the usual plane wave (3.1) is that the sign of the g_{++} term is reversed. This means that the T-dual background is a real solution of IIB* theory rather than IIB theory, as expected since we T-dualized along the lightcone. Let us call this background the IIB* plane wave. Above we denote by $\hat{F}$ a RR field in the IIB* theory, to distinguish it from the RR fields in ordinary type IIB. The IIB* plane wave is clearly related to the ordinary IIB plane wave by the analytic continuation $\mu \to i\mu$ which acts on the RR field strength as $F \to iF \equiv \hat{F}$.

[3] An alternative quantization that would avoid imaginary eigenvalues for the states built from the bosonic zero modes has recently been discussed in [16]: the inverted harmonic oscillator admits a continuous spectrum of delta-function normalizable scattering states, and one could consider those instead of the discrete states discussed here. These states lead to the same one loop amplitude as the discrete states. For our purposes one would need to extend the discussion of [16] to include the states build from the fermionic zero modes.

Now consider the action of these T-dualities on branes. A lorentzian $(+,-,m,n)$ brane in the IIB plane wave is mapped to a (m,n) brane in the IIB* plane wave. The spectrum and amplitudes of the latter will be related to those for a (m,n) brane in the IIB plane wave via analytic continuation $\mu \to i\mu$. This explains the relationship between the spectra of lorentzian branes and euclidean branes in the IIB plane wave.

3.3 Open/closed duality

Since one can describe spacelike branes in the closed channel with the usual lightcone gauge and in the open channel by the modified lightcone gauge choice, it is straightforward to check the Cardy consistency condition. Most of the ingredients required are given in the discussions of [5, 22], in particular, the modular transformations of the relevant mass deformed modular functions.

However, these papers leave open the issue of whether the cylinder condition is satisfied for branes displaced from the origin in transverse space. Indeed, it has been suggested that the lack of dynamical supersymmetry for such displaced branes could lead to a violation of the Cardy condition. Here we show that the amplitudes for displaced branes do satisfy the cylinder condition and that such displaced spacelike branes are annihilated by the same number of supercharges as the branes at the origin. Moreover, our discussion of the computation of the cylinder amplitude in the modified open string gauge involves non-trivial new features, namely the use of the canonical hamiltonian rather than the non-conserved lightcone hamiltonian.

3.3.1 Closed string channel

The relevant features of the closed string mode expansions are reviewed in the appendix. The gluing conditions (for a boundary on the worldsheet at $\tau = 0$) are

$$\begin{aligned} N: p_0^r \, |B(p^+)\rangle &= 0\,; & (\alpha_n^{1r} + \alpha_{-n}^{2r}) \, |B(p^+)\rangle &= 0\,; \\ D: x_0^{r'} \, |B(p^+)\rangle &= x^{r'} \, |B(p^+)\rangle\,; & (\alpha_n^{1r'} - \alpha_{-n}^{2r'}) \, |B(p^+)\rangle &= 0\,, \\ (\theta_0^1 + i\eta M \theta_0^2) \, |B(p^+)\rangle &= 0\,; & (\theta_{-n}^1 + i\eta M \theta_n^2) \, |B(p^+)\rangle &= 0\,. \end{aligned} \tag{3.31}$$

where N and D are Neumann and Dirichlet directions respectively. Note that $x^{r'}$ is the eigenvalue of the operator $x_0^{r'}$ and recall that in light cone gauge x^+ and x^- necessarily satisfy Dirichlet boundary conditions in the closed string channel. Here $M_{ab} = (\gamma^{r'_1} \tilde{\gamma}^{r'_2} \cdots \tilde{\gamma}^{r'_{8-p}})_{ab}$ is the product of the gamma matrices over the Dirichlet directions and $\eta = \pm 1$ describes brane and anti-brane respectively. The boundary state at general x^+ can be obtained by acting with the time evolution operator e^{-iHx^+/p^+} where H is the closed string lightcone hamiltonian given by[4]

$$H = \frac{1}{2}(p_0^2 + m^2 x_0^2) + im\theta_0^1 \Pi \theta_0^2 + \sum_{\mathcal{I}=1,2} \sum_{n>0} (\alpha_{-n}^{\mathcal{I}I} \alpha_n^{\mathcal{I}I} + \omega_n \theta_{-n}^{\mathcal{I}} \theta_n^{\mathcal{I}})\,. \tag{3.32}$$

[4]Note that the definitions of conserved charges in terms of worldsheet fields are as given in [15, appendix B]. Our conventions differ from those of [15] in that we use SO(8) rather than SO(9,1) spinors; the oscillators have also been rescaled for convenience, compare [15, appendix C] with the appendix A here.

The closed string is invariant under 16 kinematical $Q^{+1,2}$ and 16 dynamical supersymmetries $Q^{-1,2}$. Let us define complex combinations as follows

$$
\begin{aligned}
Q^+ &= Q^{+1} + i\eta M Q^{+2} , \\
Q^- &= Q^{-1} + i\eta M Q^{-2} - \frac{1}{2} i\mu\sqrt{p^+}\sum_{r'} x^{r'}\gamma^{r'} M\Pi(Q^{+1} - i\eta M Q^{+2}) .
\end{aligned} \tag{3.33}
$$

As discussed in [15], the displaced spacelike brane is annihilated by the supercharges

$$
Q^+ \left|B(p^+)\right\rangle = 0 ; \qquad Q^- \left|B(p^+)\right\rangle = 0 . \tag{3.34}
$$

i.e., the boundary state is Grassmann analytic (it is annihilated by the $Q^\pm$, but not the complex conjugates). In other words, the supercharges preserved by the boundary state form a 16 dimensional subspace of the complex space spanned by $Q^\pm, \bar{Q}^\pm$. The unusual reality conditions of the preserved supercharges are related to the fact that the corresponding worldvolume theory is spacelike.

The explicit solution for the boundary state is [14, 5]

$$
\begin{aligned}
\left|B(p^+)\right\rangle &= \mathcal{N}\exp\left(\sum_{n=1}^{\infty}(\omega_n^{-1} M_{IJ}\alpha^{I1}_{-n}\alpha^{J2}_{-n} - i\eta M\theta^1_{-n}\theta^2_{-n})\right)\left|B^0(p^+)\right\rangle \\
\left|B^0(p^+)\right\rangle &= (M_{IJ}\left|I\right\rangle\left|J\right\rangle + i\eta M_{\dot a\dot b}|\dot a\rangle|\dot b\rangle) e^{-\frac{1}{2}\sum_r a_0^r a_0^r + \frac{1}{2}\sum_{r'}\left(a_0^{r'} - i\sqrt{2m}x^{r'}\right)^2}\left|0\right\rangle ,
\end{aligned} \tag{3.35}
$$

where M_{IJ} is a matrix with diagonal entries of -1 and 1 for Neumann and Dirichlet directions respectively. The matrix $M_{\dot a\dot b} = (\tilde\gamma^{r'_1}\gamma^{r'_2}...\gamma^{r'_{8-p}})_{\dot a\dot b}$ is the product of gamma matrices in the Dirichlet directions. $\mathcal{N}$ is an overall normalization, to be fixed by factorizing the annulus computed in the open string channel. Note that the action of the fermion zero modes (or equivalently the kinematical supercharges) on these states is

$$
\sqrt{2}\theta_0^a|I\rangle = \gamma^I_{a\dot a}|\dot a\rangle , \qquad \sqrt{2}\theta_0^a|\dot a\rangle = \tilde\gamma^I_{\dot a a}|I\rangle . \tag{3.36}
$$

In tensor products such as $|I\rangle|J\rangle$ θ_0^1 and θ_0^2 act on the first and second states respectively.

This representation of the fermion zero mode part of the ground state is particularly useful for determining supergravity field sources. Consider first $M_{IJ}|I\rangle|J\rangle$: one decomposes M_{IJ} into SO(8) representations $8\otimes 8 = 35 + 28 + 1$. The 35 is the symmetric traceless part, corresponding to the transverse graviton h_{IJ}; the 28 is the antisymmetric part, corresponding to the transverse 2-form b_{IJ}, and the singlet is the dilaton ϕ. As usual, one can choose lightcone gauge for the supergravity fluctuations, $\psi_{-M\dots} = 0$, and the non-dynamical modes $\psi_{+M\dots}$ are determined in terms of these transverse modes. The RR part of the boundary state $M_{\dot a\dot b}|\dot a\rangle|\dot b\rangle$ can also be decomposed as

$$
M_{\dot a\dot b} = \frac{1}{8}\delta_{\dot a\dot b}\,\mathrm{tr}(M) + \frac{1}{16}\gamma^{IJ}_{\dot a\dot b}\,\mathrm{tr}(\gamma^{IJ}M) + \frac{1}{384}\gamma^{IJKL}_{\dot a\dot b}\,\mathrm{tr}\left(\gamma^{IJKL}M\right) , \tag{3.37}
$$

defining the couplings to the RR scalar χ, two-form c^R_{IJ} and four-form c^R_{IJKL} respectively. Again the non-propagating components $\psi_{+M\dots}$ are determined by the transverse modes.

This discussion follows that of [27] for the supergravity sources of boundary states in lightcone gauge in flat space. There is an important difference in the plane wave, however: the states $|I\rangle|J\rangle$ and $|\dot{a}\rangle|\dot{b}\rangle$ are not eigenstates of the hamiltonian. To describe the lightcone time evolution of the boundary state it is convenient to write the boundary state instead in terms of such eigenstates, constructed in [28]. To do so one defines complex combinations of fermion zero modes

$$\theta_R = \frac{1}{2\sqrt{2}}(1+\Pi)(\theta_0^1 + i\theta_0^2)\,; \qquad \theta_L = \frac{1}{2\sqrt{2}}(1-\Pi)(\theta_0^1 + i\theta_0^2)\,, \tag{3.38}$$

and chooses the closed string vacuum to be such that $\bar{\theta}_L|0\rangle = \theta_R|0\rangle = 0$. Then the boundary state for branes such that $M^2 = -1$, i.e. the $(2,0)$ and $(4,2)$ branes, is

$$\exp\left(-\frac{1}{2}\eta M_{ab}\theta_L^a\theta_L^b + \frac{1}{2}\eta M_{ab}\bar{\theta}_R^a\bar{\theta}_R^b\right)|0\rangle\ , \tag{3.39}$$

whilst an analogous expression holds for the $(1,3)$ branes for which $M^2 = 1$; we will not need the explicit expression here. Expanding the exponential, one can then infer the supergravity sources by comparison with the tables given in [28].

From the explicit form of the boundary state (3.35) it is immediately apparent that these branes source purely imaginary RR fields, which is to be expected since the branes are spacelike. Note also that the boundary states at $x^+ = 0$ source only the graviton, dilaton and (imaginary) RR p-form potential, as in flat space [29]. Boundary states at general x^+ however source different supergravity fields and are not pure position/momentum eigenstates. We will discuss later the lightcone time evolution of the branes.

The cylinder amplitude between separated pairs of branes is given by

$$\mathcal{A}(X^+, X^-, x_1, x_2) = \left\langle x_1^+, x_1^-, x_1|\Delta|x_2^+, x_2^-, x_2\right\rangle, \tag{3.40}$$

where Δ is the closed string propagator and (x_1, x_2) are the transverse positions of the branes. The branes are also separated in the lightcone directions so that $X^{\pm} = (x_2^{\pm} - x_1^{\pm})$. Fourier transforming along the lightcone one gets

$$\begin{aligned}\mathcal{A}(X^+, X^-, x_1, x_2) &= \frac{1}{2\pi i}\int dp^+ dp^- e^{ip^+X^- + ip^-X^+}\langle -p^-, -p^+, x_1|\frac{1}{p^+p^- + H}|p^-, p^+, x_2\rangle\,; \\ &= \int_0^\infty dp^+ e^{ip^+X^-}\langle -p^+, x_1|e^{-\frac{iHX^+}{p^+}}|p^+, x_2\rangle\,,\end{aligned} \tag{3.41}$$

where H is the lightcone hamiltonian given in (3.32). A suitable regularization prescription is implicit in these expressions; we will discuss in the next section the computation of the integrated amplitudes. One can rewrite this amplitude as an integration over a cylinder parameter t (with $X^+ = \pi p^+ t$, the π normalization being included for later convenience) so that

$$\mathcal{A}(X^+, X^-, x_1, x_2) = \int_0^\infty \frac{dt}{t} e^{i\frac{X^+X^-}{\pi t}}\tilde{\mathcal{A}}(t, x_1, x_2)\,, \tag{3.42}$$

where

$$\tilde{\mathcal{A}}(t, x_1, x_2) = \left\langle -p^+, x_1|e^{-i\pi Ht}|p^+, x_2\right\rangle\ . \tag{3.43}$$

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This amplitude is the same as that given in [5], except that here we allow for non-zero Dirichlet positions. Thus one may immediately write down the amplitude as

$$\tilde{\mathcal{A}}(t,x_1,x_2) = \mathcal{N}_1\mathcal{N}_2\mathcal{A}_D(1-q^m)^{\frac{p-8}{2}}\left(\frac{f_1^m(q)}{f_1^m(q)}\right)^8 , \tag{3.44}$$

where $\mathcal{A}_D$ is the part that depends on the Dirichlet zero modes (i.e. the position of the brane), $q = e^{-2\pi i t}$ and the modular function is [5]

$$f_1^m(q) = q^{-\Delta_m}(1-q^m)^{1/2}\prod_{n=1}^{\infty}(1-q^{\sqrt{m^2+n^2}}) \tag{3.45}$$

with Δ_m the Casimir energy of a boson of mass m on a cylinder with periodic boundary conditions, whose integral representation is given in [5].

The part of the amplitude that depends on the Dirichlet zero mode part is given by

$$\mathcal{A}_D = \langle 0| e^{\frac{1}{2}\left(\bar{a}_0 + i\sqrt{2m}x_1\right)^2} e^{\ln(z)a_0\bar{a}_0} e^{\frac{1}{2}\left(a_0 - i\sqrt{2m}x_2\right)^2}|0\rangle , \tag{3.46}$$

where for notational simplicity the r' indices are suppressed and $z = q^{\frac{1}{2}m}$. Up to normalization, this is the quantum mechanical amplitude $\langle x_1|\exp iHt|x_2\rangle$, where H is the hamiltonian of the harmonic oscillator. The result is well known but we present an elementary evaluation of this amplitude in appendix B; the result is

$$\mathcal{A}_D = \frac{1}{(1-z^2)^{\frac{1}{2}}} e^{-\frac{m}{1-z^2}\left(x_1^2+x_2^2-2zx_1x_2\right)} . \tag{3.47}$$

Putting this result for the Dirichlet zero modes together with the rest of the amplitude one gets

$$\mathcal{A}(X^+,X^-,x_1,x_2) = \int_0^\infty \frac{dt}{t} e^{i\frac{X^+X^-}{\pi t}} \mathcal{N}_1\mathcal{N}_2 e^{-\frac{m}{1-z^2}(x_1^2+x_2^2-2zx_1x_2)}\left(\frac{f_1^m(q)}{f_1^m(q)}\right)^8 . \tag{3.48}$$

3.3.2 Open string channel

The one loop amplitude for the open strings in the modified lightcone gauge is given by

$$Z = \int_0^\infty \frac{ds}{s}\,\mathrm{Tr}\left((-)^F e^{2i\pi H_o s}\right) , \tag{3.49}$$

where H_o is the open string canonical hamiltonian for which one now relaxes the constraint $H_o = 0$. Since the hamiltonian generates worldsheet time evolution, it is clearly the correct generator to describe a loop of open strings.

One needs, however, to be careful about the sign in the exponent. Since we are working in lorentzian signature comparison of the cylinder amplitudes between open and closed channels is subtle. When one carries out an S-transformation which exchanges the sides of a lorentzian cylinder, this also changes the overall signature. Thus we will need to compare a $(-1,1)$ signature cylinder in the closed channel with a $(1,-1)$ signature cylinder in the open channel. Our previous discussions used $(-1,1)$ signature in the open channel and the effect of the signature change is to change the overall sign in the hamiltonian,

$H_o \to -H_o$, which can be seen by double analytic continuation in τ and σ. This explains the plus sign in the exponent above. It is then convenient to rewrite the amplitude in the equivalent form

$$Z = \int_0^{-\infty} \frac{ds}{s} \operatorname{Tr}\left((-)^F e^{-2i\pi H_o s}\right), \tag{3.50}$$

i.e. changing the overall sign of s.

Evaluating this amplitude one finds

$$Z = \int_0^{-\infty} \frac{ds}{s} e^{-i\frac{\sigma_{10-p}^2}{\pi}s} (2\sinh(\mu X^+ s))^{4-p} \left(\frac{f_1^{\tilde{m}}(\tilde{q})}{f_1^{\tilde{m}}(\tilde{q})}\right)^8, \tag{3.51}$$

where now

$$f_1^{\tilde{m}}(\tilde{q}) = \tilde{q}^{-\Delta_{\tilde{m}}} (1-\tilde{q}^{i\tilde{m}})^{1/2} \prod_{n=1}^{\infty} (1-\tilde{q}^{\sqrt{n^2-\tilde{m}^2}}), \tag{3.52}$$

with $\tilde{q} = e^{-2i\pi s}$. Also σ_{10-p}^2 is given by

$$\sigma_{10-p}^2 = 2X^+X^- + \frac{\mu X^+}{\sin(\mu X^+)} \sum_{r'=p+1}^{8} \left(\cos(\mu X^+)((x_1^{r'})^2 + (x_2^{r'})^2) - 2x_1^{r'} x_2^{r'}\right). \tag{3.53}$$

This is the geodesic distance between two points in the plane wave [30], separated by $X^{\pm}$ in the lightcone directions and at $(x^r = 0, x_1^{r'})$ and $(x^r = 0, x_2^{r'})$ respectively in the transverse directions.

The integrand obtained in (3.51) derives from the following structure of the open string spectra. Whilst the number of bosonic states matches the number of fermionic states at every $\hat{H}_o$ eigenvalue for stringy states, there is a mismatch between bosons and fermions for zero modes. For $p = 2$ there is a mismatch at the first three levels of the spectrum, $\hat{H}_0 = -i\tilde{m}, 0, i\tilde{m}$, giving rise to the $\sinh^2$ factor. For $p = 4$ it is only the vacuum state which is unpaired, giving a factor of one, whilst for $p = 6$ there is a mismatch at every level in the zero mode spectrum, giving the $1/\sinh^2$ factor. A detailed discussion of the spectra in the related case of lorentzian D_- branes is given in [15].

With the conventions and normalizations used here, in the the open string channel we have a cylinder of length π and of circumference $2\pi s$ whilst in the closed string channel the circumference is 2π and the length is πt. Under the S transformation $s \to -1/t$ and one should in addition perform a conformal transformation so that the length and circumference of the cylinder are the same as before. This implies that the mass parameters are related as $\tilde{m} = imt$. This can be seen as follows. Before fixing the lightcone gauge the sigma model was (classically) conformally invariant. In the lightcone gauge $g_{++} \sim m^2(x^I)^2$ and the standard conformal transformation of g_{++} implies that m should transform. As discussed above, one needs to take into account the signature change under the S transformation, and thus the amplitudes (3.48) and (3.51) should agree when $s = -1/t$ and $\tilde{m} = imt$.

That the amplitudes do agree follows from the lorentzian S modular transformation for the mass deformed modular functions:

$$f_1^m(e^{-2\pi it}) = f_1^{imt}\left(e^{\frac{2\pi i}{t}}\right), \tag{3.54}$$

which can be derived via analytic continuation of the proof for the euclidean transformation given in [5, appendix A]. Note that the $m \to 0$ limit of this identity gives

$$f_1(e^{\frac{2\pi i}{t}}) = (it)^{1/2} f_1(e^{-2\pi i t}), \tag{3.55}$$

which is the correct lorentzian transformation property of the usual modular function.

The cylinder amplitudes then agree provided that the boundary state normalization is

$$\mathcal{N} = (2\sinh(\pi m))^{\frac{1}{2}(4-p)} e^{\frac{1}{2}m\sum_{r'}(x^{r'})^2}, \tag{3.56}$$

and we relate the open and closed string positions by $x_{\text{closed}} = \sqrt{2}x_{\text{open}}$. The latter identification follows from the overall normalizations of the (gauge fixed) open and closed string actions. The $x^{r'}$ dependent normalization reflects the fact that

$$|x^{r'}\rangle = e^{\frac{1}{2}m(x^{r'})^2} e^{\frac{1}{2}(a_0^{r'} - i\sqrt{2m}x^{r'})^2} |0\rangle \tag{3.57}$$

are the normalized position eigenstates satisfying $\langle x_1^{r'} | x_2^{r'} \rangle = \sqrt{\pi}\delta(x_1^{r'} - x_2^{r'})$. Note that a non-trivial consistency check on these normalizations is provided by the agreement between the amplitudes for general (x_1, x_2) and for different pairs of (anti)-branes Ep-Eq ($p \neq q$), but we shall not present the details here as similar computations for branes at the origin were reported in [5].

3.4 Behavior of integrated amplitudes

The cylinder amplitudes vanish for cylinders which end on the same brane; this follows from the presence of fermion zero modes. Thus the first correction to the self energy of the brane vanishes, presumably along with all higher corrections. The overlap between parallel branes at the same lightcone position but separated in the transverse directions also vanishes.

However, D_- branes of the same type but separated along the lightcone are not annihilated by the same combinations of supercharges. In the plane wave, the kinematical charges Q^+, which are represented in terms of the fermion zero modes, do not commute with the lightcone hamiltonian. Thus there is non-trivial behavior for the cylinder amplitudes between D_- branes separated along the lightcone which we now discuss. There are three cases to consider, corresponding to $(2,0)$ E2 branes; $(3,1)$ E4 branes and $(4,2)$ E6 branes.

Note that D_+ branes, i.e. (m,n) branes for which $n \neq (m \pm 2)$, are annihilated by combinations of kinematical supercharges which commute with the hamiltonian [15, 22]. This implies that the cylinder amplitudes for such branes are zero regardless of the brane separations. It is for this reason that we focus on D_--branes which better illustrate the generic behavior of branes in curved backgrounds of interest here.

Throughout this section we focus on the behavior of amplitudes for generic brane separations. In the next section we identify and discuss the physical interpretation of distinguished separations for which the amplitudes take special values.

E2-branes. In this case the cylinder amplitude is

$$Z = \int_0^\infty \frac{ds}{s} e^{i\frac{\sigma_8^2}{\pi}s} (2\sinh(\mu X^+ s))^2 \,. \tag{3.58}$$

The integral can be computed by analytically continuing $x^+ \to x_E^+ = ix^+$; the integral is then convergent provided that x^- is positive. Evaluating the integral under these conditions and analytically continuing the answer to real x^+ and general values of x^- one obtains

$$Z = -4\ln\left(1 + \left(\frac{2\pi\mu X^+}{\sigma_8^2}\right)^2\right) . \tag{3.59}$$

Note that in this case the integral is convergent at the lower end $s \to 0$.

E4-branes. In this case the cylinder amplitude is

$$Z = \int_0^\infty \frac{ds}{s} e^{i\frac{\sigma_6^2}{\pi}s} , \tag{3.60}$$

which is clearly non convergent at both ends of the integration. Analytic continuation can remove the $s \to \infty$ divergence, but not the one for the small s. This divergence must be regulated by cutting off the integral at $s = \Lambda$. The regulated amplitude is thus

$$Z = \Gamma\left(0, -\frac{i\Lambda\sigma_6^2}{\pi}\right) \equiv E_1\left(-\frac{i\Lambda\sigma_6^2}{\pi}\right) , \tag{3.61}$$

where $\Gamma(k, x)$ is the incomplete Gamma function, with $\Gamma(0, x)$ equivalent to the exponential integral $E_1(x)$. Expanding this for small x:

$$Z = \ln\left(-\frac{i\Lambda\sigma_6^2}{\pi}\right) - \gamma + \cdots , \tag{3.62}$$

where γ is the Euler constant and the ellipses denote terms which vanish as $\sigma_6^2\Lambda \to 0$.

E6-branes. The integral to be evaluated is

$$Z = \int_0^\infty \frac{ds}{s} e^{i\frac{\sigma_4^2}{\pi}s} (2\sinh(\mu X^+ s))^{-2} , \tag{3.63}$$

which is clearly convergent at the upper end of the integration but divergent at the lower end. In this case computing the exact integral is difficult and thus we compute only the divergent parts which can obtained from expanding the integrand for small s:

$$Z = \int_\Lambda^\infty \frac{ds}{s} e^{i\frac{\sigma_4^2 s}{\pi}} \left(\frac{1}{4(\mu X^+ s)^2} - \frac{1}{12} + \cdots\right) , \tag{3.64}$$

where the ellipses denote finite terms. The divergent terms are thus

$$\begin{aligned} Z &= \frac{1}{(2\mu X^+)^2}\Gamma\left(-2, -\frac{i\Lambda\sigma_4^2}{\pi}\right) - \frac{1}{6}\Gamma\left(0, -\frac{i\Lambda\sigma_4^2}{\pi}\right) + \cdots \\ &= \frac{1}{(2\mu X^+)^2}\left(\frac{1}{2\Lambda^2} + \frac{i\sigma_4^2}{\pi\Lambda} + \frac{\sigma_4^4}{2\pi^2}\ln\left(-\frac{i\Lambda\sigma_4^2}{\pi}\right)\right) - \frac{1}{12}\ln\left(-\frac{i\Lambda\sigma_4^2}{\pi}\right) + \cdots \end{aligned} \tag{3.65}$$

3.4.1 Long cylinder divergences

As is well-known, the amplitude in the long cylinder limit should be equal to exchange diagram of massless closed string modes. It was recently verified in [29] that the amplitudes in the plane wave do exhibit this behavior. Let us briefly review the field theory computation. To compute the exchange diagram one needs the quadratic part of the supergravity action and the D-brane couplings. These have the form

$$S = \frac{1}{4\kappa^2}\int d^{10}x\left(\psi^\dagger(\Box - 2i\mu c\partial_-)\psi + \delta^{10-p}(x-x_0)\lambda(\psi+\eta\bar{\psi})\right) \tag{3.66}$$

for a euclidean p-brane, where ψ denotes any supergravity field (we suppress all indices). (c,λ) are parameters that depend on the specific gauge fixed fluctuation under consideration whilst $\eta = \pm 1$ for brane/anti-brane respectively. c derives from the lightcone mass of the fluctuation in the supergravity action and λ (which is proportional to the brane tension T_p) from the source in the DBI action.

The explicit constants for all cases of interest can be found in [29]. Note in particular that there has to be a relative factor of i between NS-NS and RR field sources because the brane is spacelike and the corresponding DBI action is analytically continued with respect to the standard lorentzian brane action.

Each mode of a given (c,λ) contributes to the exchange between two separated branes. and it was shown in [29] that the total exchange was

$$Z_p = -4\pi(4\pi^2)^{4-p}\sin^4(\mu X^+)G^{10-p}(X^+,X^-,x_1^{r'},x_2^{r'})\,, \tag{3.67}$$

where we have used the fact that $T_p^2\kappa^2 = \pi(4\pi^2)^{4-p}$ in our conventions. Here the branes have lightcone separations (X^+,X^-) and transverse positions $(x_1^{r'},x_2^{r'})$ respectively. G^{10-p} is the propagator for a massless scalar (i.e. $\Box\psi = 0$) over the $(10-p)$ dimensions transverse to the brane which is given by integrating over the worldvolume directions the $10d$ propagator [30]:

$$G^{10}(X^+,X^-,x_1^I,x_2^I) = \frac{(\mu X^+)^4}{4\pi^5\sin^4(\mu X^+)}\int_0^\infty \frac{dk}{k^5}e^{\frac{i\sigma_{10}^2}{k}}\,, \tag{3.68}$$

where σ_{10}^2 is the 10d geodesic separation given in (3.53). Integrating (3.68) (again in the convergent regime with imaginary x^+ and then analytically continuing) one gets

$$G^{10}(X^+,X^-,x_1^I,x_2^I) = \frac{3(\mu X^+)^4}{2\pi^5\sin^4(\mu X^+)}\frac{1}{\sigma_{10}^8}\,. \tag{3.69}$$

Note that the limit $\mu \to 0$ is clearly smooth, and reproduces the usual propagator in Minkowski space.

As is very familiar, integrating either (3.68) (or equivalently (3.69)) over the first worldvolume direction in flat space gives an overall volume factor following from translational invariance:

$$G^9_{\text{flat}}(X^+,X^-,x_1^{r'},x_2^{r'}) \sim \int dx_1^1 dx_2^1 \frac{1}{(\sigma_9^2+(x_1^1-x_2^1)^2)^4} \sim V_1\frac{1}{(\sigma_9^2)^{7/2}}\,, \tag{3.70}$$

where V_1 is the regulated length and σ_9^2 is the 9d geodesic separation. Thus for the field theory exchange between spacelike p-branes in flat space one finds the usual

$$Z_{\rm flat} \sim \frac{V_p}{(\sigma_{10-p}^2)^{(8-p)/2}}\,, \tag{3.71}$$

where the overall prefactor is of course zero for brane/brane field exchange because of supersymmetric cancellation.

In the plane wave, translations in the x^I directions act as

$$\delta x^- = \mu \sin \mu x^+ \epsilon^I x^I, \qquad \delta x^I = \cos \mu x^+ \epsilon^I\,. \tag{3.72}$$

This implies that the system of branes separated along the lightcone directions is not generically invariant under translations in the x^I directions and it leads to very different behavior for the integrated propagators compared to flat space: there is no overall volume factor and the power law behavior is modified. Thus

$$Z_p = -2^{8-2p}(\pi\mu X^+)^{4-p}\int_0^\infty \frac{dk}{k^{5-p}} e^{i\frac{\sigma_{10-p}^2}{k}}\,, \tag{3.73}$$

which clearly reproduces the small s or equivalently large t behavior of the integrands in the string amplitudes (3.48) and (3.51) for $p=2$ and $p=4$. For the E2-brane one gets the finite answer given in [29]

$$Z_{p=2} = -2^4\frac{(\pi\mu X^+)^2}{(\sigma_8)^4}\,, \tag{3.74}$$

in agreement with the large σ^2 behavior of the string amplitude. For $p=4$ the field theory amplitude is clearly exactly the string amplitude (3.60) and reproduces its logarithmic divergence.

For $p=6$ the expression (3.73) is no longer valid: (3.73) was obtained by exchanging the order for the integrations over k and x^r respectively which is only permitted when the integrals are convergent. However, one can straightforwardly integrate (3.69) over the worldvolume coordinates to reproduce the divergent parts of the string amplitude given in (3.64).

We have been discussing the field theory exchange between two parallel separated branes. In flat space the long range supergravity fields sourced by a single brane are translationally invariant along the Neumann directions and are proportional to the relevant propagator $1/\sigma_{10-p}^{8-p}$, i.e. (3.71) without the overall volume factor.

In the plane wave the absence of translational invariance means that the long range supergravity field sourced by the brane depends on the position in the Neumann directions. The explicit behavior is given by integrating the propagator over the worldvolume directions. For the behavior of a massless supergravity mode ψ (i.e. $c=0$) at a given point far from the brane one gets

$$\psi \sim (\mu X^+)^{4-\frac{1}{2}p}\frac{\tan^{\frac{1}{2}p}(\mu X^+)}{\sin^4(\mu X^+)}\hat{\sigma}^{p-8}\,, \tag{3.75}$$

where

$$\hat{\sigma}^2 = 2X^+X^- - \mu X^+ \tan\left(\mu X^+\right) \sum_r (X^r)^2 + \\ + \frac{\mu X^+}{\sin(\mu X^+)} \sum_{r'} \left(((x_b^{r'})^2 + (X^{r'})^2) \cos(\mu X^+) - 2x_b^{r'} X^{r'} \right) , \quad (3.76)$$

and $x_b^{r'}$ is the brane position, X^I is the transverse position of the observation point and (X^+, X^-) is the lightcone separation between the brane and the observation point. Thus the power law dependence is the same as in flat space but the field sourced is not translationally invariant along the directions parallel to the brane. Integrating with respect to X^r gives the brane/brane exchange behavior, again demonstrating that the logarithmic and power law divergences discussed above result from the infinite volumes of the branes.

3.5 Distinguished x^+ separations and plane wave geodesics

In the previous section we have discussed features of the amplitudes for generic separations, emphasizing the lack of translational invariance. For special brane separations, however, translational invariance is restored. This happens when

$$\mu X^+ = l\pi , \qquad l \in \mathbb{Z} . \quad (3.77)$$

In this case, (3.72) yields $\delta x^- = 0$, $\delta x^I = (-1)^l \epsilon^I$ and one expects the amplitudes to become similar to the flat space amplitudes.

Physically one can understand these distinguished values as arising from the behavior of geodesics in the plane wave: a generic geodesic will reconverge to the same transverse position x^I after evolution by $\mu X^+ = 2\pi$. Labelling the geodesic by X^+, its trajectory is [30]–[32]

$$x^-(X^+) = x_1^- + \frac{1}{4}(x_1^2 - p_1^2)\sin(2\mu X^+) - \frac{1}{2} x_1 \cdot p_1 \cos(2\mu X^+) + CX^+ + \frac{1}{2} x_1 \cdot p_1 ; \\ x^I(X^+) = x_1^I \cos(\mu X^+) + p_1^I \sin(\mu X^+) , \quad (3.78)$$

where (x_1^-, x_1^I, p_1^I) are initial conditions for the geodesic. The constant C is also given in terms of initial conditions via $p_1^- + \frac{1}{2}\mu(p_1^2 - x_1^2)$.

Thus a generic geodesic will reconverge to its original transverse position x_1^I after $\mu X^+ = \pi l$ with l even and it will pass through $-x_1^I$ for l odd. After evolution through $\mu X^+ = \pi l$, the geodesic will be shifted in x^- by an amount $\pi l C/\mu$. Such focusing is unavoidable given finite valued initial conditions for the geodesic; one can only avoid the focusing with infinite initial velocities for the geodesic (i.e. p_1 is infinite).

The focusing of geodesics can be understood in terms of focusing of geodesics on $AdS_5 \times S^5$. As was reviewed in section 2, there is a focusing of geodesics between the north and south poles of the S^5 and after $t = \pi$ on global AdS_5. Now, recall that in taking the Penrose limit one defines coordinates

$$\mu x^+ = \frac{1}{2}(\tau + \theta), \qquad x^- = \frac{\mu R^2}{4}(\theta - \tau) \quad (3.79)$$

and then takes the limit $R^2 \to \infty$. Clearly for x^- to stay finite in the limit we need

$$\theta = \tau + \mathcal{O}\left(\frac{1}{R^2}\right) \tag{3.80}$$

so the geodesics that join focal points of AdS_5 *and* of the circle of S^5 along which we boost survive in the limit. These are the geodesics discussed above.

Note that (3.53) implies that the geodesic distance becomes infinite for $\mu X^+ = l\pi$ if $(x_1^I + (-1)^{l+1} x_2^I) \neq 0$ even if the latter is finite. To regulate the geodesic distance we consider the x^+ separation to be given by

$$\mu X^+ = l\pi + \epsilon\,, \qquad l \in \mathbb{Z}\,, \tag{3.81}$$

where ϵ is infinitesimal. The geodesic distance becomes

$$\sigma_{10}^2 = \sum_{I=1}^{8}\left(\frac{\pi l}{\epsilon}(x_1^I + (-1)^{l+1}x_2^I)^2 + \left(\frac{2\pi l X^-}{\mu} + (x_1^I + (-1)^{l+1}x_2^I)^2\right)\right) + \mathcal{O}(\epsilon), \tag{3.82}$$

which clearly shares the translational invariance of the flat space geodesic distance. Now consider the behavior of the massless field propagator: from (3.69) one sees that it is finite as $\epsilon \to 0$, and is exactly the same as in flat space. Integrating over the worldvolume coordinates now yields the following expression for the total field theory exchange

$$Z_p \sim -l^{4-\frac{1}{2}p} V_p \epsilon^{\frac{1}{2}p} \int_0^\infty \frac{dk}{k^{5-\frac{1}{2}p}} e^{i\frac{\sigma_{10-p}^2}{k}} \sim -\frac{V_p \epsilon^4}{(\sum_{r'}(x_1^{r'} + (-1)^{l+1}x_2^{r'})^2)^{4-\frac{1}{2}p}}, \tag{3.83}$$

where V_p is the regulated brane volume and overall numerical factors are suppressed. This expression explicitly demonstrates the reinstated translational invariance. Moreover the total amplitude for field theory exchange vanishes as $\epsilon \to 0$.

The above implicitly assumes that the brane separations in the $x^{r'}$ Dirichlet directions are such that the geodesic distance between the branes diverges in the limit $\mu X^+ = l\pi$. When the brane positions take the special values $x_1^{r'} = (-)^l x_2^{r'}$ the geodesic separation remains finite in this limit. As we have discussed, for these separations there are an infinite number of geodesics connecting the two branes along each such Dirichlet direction rather than a unique geodesic as is usually the case.

Suppose that $x_1^{r'} = (-)^l x_2^{r'}$ for j of the Dirichlet directions, i.e. the branes are either coincident in these directions or their positions are reflections of one another; the amplitude in this limit can conveniently be obtained by taking the limit of the exponential in the integrand in (3.83) as $x_1^{r'} \to (-)^l x_2^{r'}$ using the identity (here and in subsequent expressions in this section integrals are implicitly computed in the convergent regime and then analytically continued; overall phase factors are suppressed)

$$\delta(x) = \lim_{\alpha\to\infty}\left(\frac{\alpha}{\sqrt{\pi}} e^{-\alpha^2 x^2}\right), \tag{3.84}$$

along with $2\pi\delta(x=0) = \tilde{V}$, where $\tilde{V}$ is the volume of momentum space[5]. The amplitude then becomes

$$Z_p \sim -\frac{V_p \tilde{V}_j \epsilon^4}{(\sum_{r'=p+j+1}^{8-p}(x_1^{r'} + (-)^{l+1} x_2^{r'})^2)^{4-\frac{1}{2}(p+q)}}, \tag{3.85}$$

where $\tilde{V}_j$ is the regulated momentum space volume for the j Dirichlet directions. Note that this expression is valid only for $j < (8-p)$; in the limit $j = (8-p)$ one obtains

$$Z_p \sim V_p \tilde{V}_{8-p} \epsilon^4 \int_0^{1/\Lambda} \frac{dk}{k} e^{\frac{2\pi i l X^-}{\mu k}} \sim V_p \tilde{V}_{8-p} \epsilon^4 \ln\left(l X^- \frac{\Lambda}{\mu}\right), \tag{3.86}$$

where Λ is a regulator.

The key features of both expressions (3.85) and (3.86) are that the amplitude scales with the Dirichlet volume and vanishes in the limit $\epsilon \to 0$. We now consider these distinguished separations in the open and closed string channels; this will clarify the physical origin of both these features.

3.5.1 Open string channel

Let us consider the limit $\mu r^+ \to 1$ in the open string mode expansions given previously. For notational ease we discuss the specific case $l = 1$ but general l follows straightforwardly from this case. In this limit the frequencies of the first stringy modes approach zero and one finds that

$$\begin{aligned}
x^r(\sigma,\tau) &= \frac{1}{\sqrt{2}}(X_0^r + P_0^r \tau)\cos(\sigma) + \cdots & (3.87)\\
x^{r'}(\sigma,\tau) &= x_1^{r'}\cos(\sigma) + \frac{1}{\sqrt{2}}(X_0^{r'} + P_0^{r'}\tau)\sin(\sigma) + \cdots & (3.88)\\
\sqrt{r^+}\theta^1 &= (M\Pi\cos(\sigma) - i\sin(\sigma))(\theta_1 - \theta_{-1}) + \cdots & (3.89)\\
\sqrt{r^+}\theta^2 &= (M^t\sin(\sigma) - i\Pi\cos(\sigma))(\theta_1 - \theta_{-1}) + \cdots\,, & (3.90)
\end{aligned}$$

where the ellipses denote unaffected terms in the mode expansions (i.e. $n^2 \neq 1$). The associated commutation relations are

$$[X_0^r, P_0^s] = i\delta^{rs}\,, \qquad [X_0^{r'}, P_0^{s'}] = i\delta^{r's'}\,, \qquad \{\theta_1, \theta_{-1}\} = \frac{1}{4}\,. \tag{3.91}$$

The Dirichlet solution is rather special, in that one loses an integration constant in this limit because the second "zero mode" solution coincides with the limit of the stringy mode solution. The string is forced to have its endpoints at $\pm x_1^{r'}$; this fits with the geodesic behavior, in that only infinite proper length geodesics will give $x_2^{r'} \neq -x_1^{r'}$. We can thus only consider this latter case when the geodesic distance is regulated via $\tilde{m} = (1+\epsilon/\pi)$ as in the previous discussion.

Computing the contributions to the conserved charge $\hat{H}_o$ from these modes one finds

$$\hat{H}_o = i\left(\sum_{r=1}^{p} a_0^r \bar{a}_0^r - 2i\theta_0 M\Pi\theta_0 + \frac{1}{2}p\right) + \frac{1}{2}\sum_I (P_0^I)^2 + \sum_{n>1}\omega_n(a_{-n}^I a_n^I + 4\theta_n\theta_{-n})\,. \tag{3.92}$$

[5]This can be seen using $\delta(x) = \int dp/(2\pi) e^{ipx}$, so $2\pi\delta(0) = \int dp$.

Thus fermion modes $\theta_{\pm 1}$ drop out of the charge in this limit since their frequencies are zero. States can now be labelled simultaneously by their $\hat{H}$ eigenvalue and by their continuous "momentum" P_0^I. Computing the annulus using this hamiltonian we obtain

$$Z = \frac{V_p V_{8-p}}{(2\pi)^8} \int \frac{ds}{s^5} e^{2iX^- s/\mu} (2\sinh(\pi s))^{4-p} (1-1)^8, \tag{3.93}$$

where V_p and V_{8-p} are the regulated volumes of the Neumann and Dirichlet directions respectively, the volume factors originating from the standard identity $\mathrm{Tr}(e^{-\pi P^2 s}) = V/(4\pi^2 s)^{\frac{1}{2}}$. Note that the volume appearing here is the position space volume. The unbalanced massive zero mode harmonic oscillators give the same contribution as for generic X^+ but there is now an overall factor of $(1-1)^8$ from the massless fermionic modes, which annihilates the amplitude.

The existence of continuous modes in the Neumann directions reflects the reinstated translational invariance along the worldvolume already noted and leads to the V_p factor. There are also continuous modes in the Dirichlet directions which leads to the V_{8-p} volume factor; from the mode expansions, one can see that these follow directly from the infinite family of geodesics connecting $x^{r'}$ and $-x^{r'}$ when $\mu X^+ = \pi$.

Note that the scaling of the amplitude with the volume is also a feature of the brane/antibrane amplitude, which can be shown to be

$$Z = \frac{V_p V_{8-p}}{(2\pi)^8} \int \frac{ds}{s^5} e^{2iX^- s/\mu} (2\sinh(\pi s))^{4-p} \left(\frac{f_4^1(\tilde{q})}{\tilde{h}_1^1(\tilde{q})} \right)^8, \tag{3.94}$$

where the function $\tilde{h}_1^1(\tilde{q})$ is related to the modular function $f_1^{\tilde{m}}(\tilde{q})$ as follows

$$\lim_{\tilde{m} \to 1+\epsilon/\pi} f_1^{\tilde{m}}(\tilde{q}) = \sqrt{8\pi\epsilon s}\, \tilde{h}_1^1(\tilde{q}) + \mathcal{O}(\epsilon) \tag{3.95}$$

and is given by

$$\tilde{h}_1^1(\tilde{q}) = \tilde{q}^{-\Delta_1} (1-\tilde{q}^i)^{\frac{1}{2}} \prod_{n=2}^{\infty} \left(1 - \tilde{q}^{\sqrt{n^2-1}}\right). \tag{3.96}$$

The modular function $f_4^1(\tilde{q})$ is given by the modular function of [5]

$$f_4^{\tilde{m}}(\tilde{q}) = \tilde{q}^{-\Delta'_{\tilde{m}}} \prod_{n=1}^{\infty} \left(1 - \tilde{q}^{\sqrt{(n-\frac{1}{2})^2 - \tilde{m}^2}}\right) \tag{3.97}$$

in the particular limit $\tilde{m} \to 1$.

Now consider the behavior of the brane/brane amplitude (3.51) computed for generic separations as one takes the limit $\tilde{m} = 1 + \epsilon/\pi$. The relevant behavior is clearly that of the first stringy modes and thus

$$Z \sim \epsilon^{\frac{1}{2}(p+j)} V_p V_j \int_0^{\infty} \frac{ds}{s} e^{i\frac{\sigma^2_{10-p-j}}{\pi}s} (2\sinh(\pi s))^{4-p} s^{\frac{1}{2}(p+j)} \left(\frac{s\tilde{h}_1^1(\tilde{q})}{s\tilde{h}_1^1(\tilde{q})} \right)^8, \tag{3.98}$$

where $x_1^{r'} = -x_2^{r'}$ for j of the Dirichlet directions. (We left the factors of s with $\tilde{h}_1^1$ for later convenience). The volume factors in the Neumann and Dirichlet directions arise from the limiting behavior of the first stringy modes, namely

$$N : \lim_{\omega_1 \to 0} \mathrm{Tr}\, e^{-2\pi i s \omega_1 a_1 a_{-1}} \to \mathrm{Tr} e^{-i\pi P_0^2} \sim \frac{V}{\sqrt{s}} ;$$
$$D : \lim_{\omega_1 \to 0} \left(\lim_{x_1 \to -x_2} \left(e^{i(x_1+x_2)^2 s/\epsilon} \mathrm{Tr} e^{-2\pi i s \omega_1 a_1 a_{-1}} \right) \right) \to \mathrm{Tr}\, e^{-i\pi P_0^2} \sim \frac{V}{\sqrt{s}} . \tag{3.99}$$

The corresponding amplitude for brane/anti-brane is given by replacing the factors of $s\tilde{h}_1^1(\tilde{q})$ in the numerator by $f_4^1(\tilde{q})/\sqrt{\epsilon}$; the result for the limiting behavior of the amplitude then clearly agrees with (3.94). Comparison of the brane/brane amplitudes (3.93) and (3.98) is slightly less clear, since they both tend to zero in this limit; they do however agree if one identifies the massless fermion contributions via $s^8\epsilon^4 \to (1-1)^8$.

3.5.2 Closed string channel

Distinguished values of X^+ are also visible in the closed string description. Looking at the annulus in the closed channel (3.48), however, it is apparent that the special behavior derives from the zero (supergravity) modes, in contrast to the open channel where for $\mu X^+ \to l\pi$ the lth stringy modes become massless. The amplitude (3.48) in the limit $\mu X^+ = l\pi + \epsilon$ becomes

$$\mathcal{A} \sim V_p \tilde{V}_j \epsilon^{\frac{1}{2}(p+j)} l^{\frac{1}{2}(p-j)} \int_0^\infty \frac{dt}{t^{1+\frac{1}{2}(p-j)}} e^{\frac{i\sigma_{10-p-j}^2}{\pi t}} \left(\sinh\left(\frac{\pi l}{t}\right)\right)^{4-p} \left(\frac{h_1^l(q)}{h_1^l(q)}\right)^8 , \tag{3.100}$$

where the function $h_1^l(q)$ is defined by the limit

$$\lim_{m \to (l+\epsilon/\pi)/t} f_1^m(q) = \sqrt{2i\epsilon} h_1^l(q) + \mathcal{O}(\epsilon) \tag{3.101}$$

and is given by

$$h_1^l(q) = q^{-\Delta_{l/t}} \prod_{n=1}^\infty (1 - q^{\sqrt{n^2+l^2/t^2}}) . \tag{3.102}$$

In the amplitude (3.100) the brane separations are again such that $x_1^{r'} = (-)^l x_2^{r'}$ for j of the Dirichlet directions and V and $\tilde{V}$ denote position space and momentum space volumes respectively. In this case the volume factors arise from the limits

$$N : \lim_{p\to 0} \left(\lim_{mt \to l} \langle -p | e^{-2\pi i t H} | p \rangle \right) \sim \lim_{p \to 0} \left(\lim_{\epsilon \to 0} \frac{1}{\sqrt{\epsilon}} e^{\frac{2ilpt}{\epsilon}} \right) \sim l^{\frac{1}{2}} t^{-\frac{1}{2}} V ;$$
$$D : \lim_{x_1 \to (-)^l x_2} \left(\lim_{mt \to l} \langle x_1 | e^{-2\pi i t H} | x_2 \rangle \right) \sim \lim_{x_1 \to (-)^l x_2} \left(\lim_{\epsilon \to 0} \frac{1}{\sqrt{\epsilon}} e^{\frac{il(x_1 - (-)^l x_2)}{\epsilon t}} \right) \sim l^{-\frac{1}{2}} t^{\frac{1}{2}} \tilde{V} . \tag{3.103}$$

The amplitude reproduces in the large t long cylinder limit the previous expression for the supergravity field theory exchange (3.85).

The open/closed duality between the two expressions (3.98) and (3.100) does not follow trivially from the previous proof for generic brane separations. The h functions inherit modular transformation properties from the f functions, namely $h_1^1(q) = s\tilde{h}_1^1(\tilde{q})$, which ensures that the Neumann and fermion contributions to the integrands in (3.98) and (3.100) agree upon modular transformation. For the Dirichlet modes, however, there is a subtlety in that the volume appearing in (3.98) is that of $X_0^{r'}$ whilst that in (3.100) is that of $p_0^{r'}$. There is a simple way to relate these quantities. Consider a cylindrical worldsheet. In the open channel the Dirichlet zero modes on the cylinder are

$$x^{r'}(\sigma) = x_1^{r'} \cos(\sigma) + \frac{1}{\sqrt{2}} X_0^{r'} \sin(\sigma)\,. \tag{3.104}$$

Note that the $P_0^{r'}\tau$ modes present for the tree level open strings are absent on the cylinder since they are not consistent with periodicity in time; this is the origin of the $\delta(P_0^{r'}=0) \sim V_{X_0^{r'}}$ factor in the annulus. In the closed channel the Dirichlet zero modes are

$$x^{r'}(\tau) = x_0^{r'} \cos(m\tau) + m^{-1} p_0^{r'} \sin(m\tau)\,. \tag{3.105}$$

Under open/closed duality the two sides of the cylinder are exchanged, $\sigma \leftrightarrow m\tau$, as discussed earlier. The matching of the cylinders in the two channels thus requires the identification $X_0^{r'} \sim m^{-1} p_0^{r'} \sim t p_0^{r'}$ and thus $V_j \sim t^j \tilde{V}_j$. This identification ensures that the Dirichlet contributions to the amplitudes (3.98) and (3.100) agree upon modular transformation.

The distinguished brane separations are also immediately apparent when one considers the hamiltonian evolution of the boundary state. A boundary state at general x_0^+ can be obtained from that at $x_0^+=0$ by acting with the evolution operator $e^{-iHx_0^+/p^+} \equiv e^{-iH\tau_0}$. It can equivalently be obtained by writing down gluing conditions for a worldsheet boundary at $-\tau_0$. The latter are

$$\begin{aligned}
(p_0^r + m x_0^r \tan(m\tau_0)) \left|B(p^+)\right\rangle_{\tau_0} &= 0\,;\\
(\alpha_n^{1r} + e^{-2i\omega_n\tau_0} \alpha_{-n}^{2r}) \left|B(p^+)\right\rangle_{\tau_0} &= 0\,;\\
\left(x_0^{r'} - m^{-1}\tan(m\tau_0) p_0^{r'} - \frac{x^{r'}}{\cos(m\tau_0)}\right) \left|B(p^+)\right\rangle_{\tau_0} &= 0\,;\\
\left(\alpha_n^{1r'} - e^{-2i\omega_n\tau_0}\alpha_{-n}^{2r'}\right) \left|B(p^+)\right\rangle_{\tau_0} &= 0\,,\\
(\cos(2m\tau_0)\theta_0^1 + (i\eta M - \Pi\sin(2m\tau_0))\theta_0^2) \left|B(p^+)\right\rangle_{\tau_0} &=\\
= (\theta_{-n}^1 + i\eta M e^{-2i\omega_n\tau_0}\theta_n^2) \left|B(p^+)\right\rangle_{\tau_0} &= 0\,.
\end{aligned} \tag{3.106}$$

From these expressions one sees that the action of the time evolution on the stringy modes is rather unimportant, a phase rotation, so the boundary state is

$$\left|B(p^+)\right\rangle_{\tau_0} = \mathcal{N} \exp\left(\sum_{n=1}^{\infty} e^{-2i\omega_n\tau_0}\left(\omega_n^{-1} M_{IJ}\alpha_{-n}^{I1}\alpha_{-n}^{J2} - i\eta M \theta_{-n}^1\theta_{-n}^2\right)\right) \left|B^0(p^+)\right\rangle_{\tau_0}\,, \tag{3.107}$$

where $\left|B^0(p^+)\right\rangle_{\tau_0}$ is the zero mode part. The explicit solution for the bosonic zero mode part of the boundary state is

$$\exp\left(-\frac{1}{2}\sum_r e^{-2i\mu x_0^+} a_0^r a_0^r + \frac{1}{2}\sum_{r'}(e^{-i\mu x_0^+} a_0^{r'} - i\sqrt{2m}\cos(\mu x_0^+) x^{r'})^2\right)|0\rangle \tag{3.108}$$

whilst following (3.38), (3.39) the fermion zero mode part of the boundary state for E2 and E6 branes (for which $M^2 = -1$) is given by

$$\exp\left(-\frac{1}{2}\eta(e^{2i\Pi\mu x_0^+}M)_{ab}\theta_L^a\theta_L^b + \frac{1}{2}\eta(e^{-2i\Pi\mu x_0^+}M)_{ab}\bar{\theta}_R^a\bar{\theta}_R^b\right)|0\rangle\,, \tag{3.109}$$

whilst that for E4 branes is slightly different but analogous (since in this case $M^2 = 1$).

The physical interpretation is the following. The lightcone hamiltonian describes the past and future evolution of a boundary state defined at some given x_0^+, which can be fixed to zero via translational invariance. Initially the Neumann and Dirichlet boundary conditions, $\partial_\tau X^r = 0$ and $X^{r'} = x^{r'}$ respectively, imply that the state is of zero momentum in the r directions and at fixed position in the r' directions.

As the state evolves in x^+, however, the source effectively rotates in the x^I directions, the time evolution reflecting the behavior of the geodesics. Consider a Neumann direction: the boundary state is a zero momentum eigenstate, as is usual for a Neumann direction, for $\mu x_0^+ = l\pi$ but a zero position eigenstate for $\mu x^+ = l\pi/2$ for l odd. In between it is a mixed eigenstate, neither pure position nor pure momentum. Similarly the Dirichlet directions are pure position eigenstates at $\mu x_0^+ = l\pi$ but pure momentum eigenstates at $\mu x_0^+ = l\pi/2$ for l odd and mixed eigenstates in between these values. There is an analogous periodicity in the fermion zero modes: the boundary state is annihilated by the same combination of zero modes after evolution through $\mu x_0^+ = l\pi$ and by precisely the opposite combination of zero modes after evolution through $\mu x_0^+ = l\pi/2$ with l odd.

Thus there is effectively a worldvolume transmutation. Take for instance the case of $(2,0)$ branes. After evolution through $\mu x^+ = l\pi/2$ with l odd the source becomes localized in the $(1,2)$ directions but uniformly distributed over the other 6 directions transverse to the lightcone: the brane is effectively an E6 brane. Note however that the evolved state for the E2 brane coincides with the initial state for the E6 brane only in zero modes; the stringy mode parts are different. Similarly the $(3,1)$ E4 brane after evolution by $\mu x^+ = l\pi/2$ becomes a $(1,3)$ E4 brane.

One should contrast this behavior with that of the analogous branes in flat space: taking $m \to 0$ in (3.106) one sees that the Neumann and fermion zero mode conditions are independent of τ_0, since they commute with the hamiltonian, but the Dirichlet zero mode condition gives

$$\left(x_0^{r'} - \tau_0 p_0^{r'} - x^{r'}\right)\left|B(p^+)\right\rangle_{\tau_0} = 0\,, \tag{3.110}$$

which describes a position eigenstate at $\tau_0 = 0$ but a zero momentum eigenstate as $\tau_0 \to \pm\infty$. Thus an initial source localized at some Dirichlet position totally disperses over the Dirichlet directions in the far future and past. The difference in the plane wave is that the effective harmonic oscillator potential prevents the source from dissipating, and causes it to recollapse at periodic intervals in x^+. It would be interesting to analyze whether these branes present interesting cosmological models for cyclic universes in the context of braneworld scenarios.

3.6 Comments on lorentzian branes

One might wonder why the spacelike branes in the plane wave rather than the usual lorentzian branes have been used to illustrate generic properties of branes in curved backgrounds. The reason is that the properties under discussion here are only visible in the plane wave for objects separated in the x^+ direction. The amplitudes for lorentzian branes at leading order are the same as those for branes in flat space [33, 29]. Let us briefly review this argument. (See also [34] for a related discussion for the annulus of branes in flat space carrying traveling waves.) The cylinder amplitude between parallel separated lorentzian branes is

$$Z = V_{+-} \int_0^\infty \frac{ds}{s} \int dp^+ dp^- \; \mathrm{Tr} \left((-)^F e^{-i(p^+p^- + H)s} \right) , \tag{3.111}$$

where V_{+-} is the regulated volume of the lightcone and H is the hamiltonian for a lorentzian Dp brane in standard lightcone gauge $x^+ = p^+\tau$. Carrying out the p^- integration gives $\delta(p^+ s)$ which enforces the limit $H_{p^+ \to 0}$ i.e. all m dependence drops out and the hamiltonian is the same as in flat space. Thence the overall amplitude is exactly as in flat space, zero because of the (now massless) fermion zero modes.

In particular, the cylinder amplitude vanishes even for branes displaced from the origin in the plane wave. Such branes admit dynamical supersymmetries in their spectra which are not expected to be preserved by interactions [18]. The supersymmetries of the spectra along with the projection onto $p^+ \to 0$ lead to the vanishing of the cylinder amplitude. It is possible, however, that the self amplitude for these branes at the next order (g_s) is non-trivial and that it develops an imaginary part corresponding to the decay of these branes, presumably back to the origin in the plane wave. Even if this is the case, these branes are certainly stable in perturbation theory because their decay time is at least of order $1/g_s$.

One should take with some caution the arguments given above for the vanishing of the cylinder, since the integral has projected onto states with $p^+ = 0$ which are of course precisely those which are inaccessible in lightcone gauge. However, independent confirmation for these arguments comes from considering the field theory limit of the exchange. Following the arguments of the previous sections, one needs to integrate the 10d propagator over the worldvolume directions, which now include the lightcone. Integrating the propagator over x^- clearly projects onto $x^+ = 0$, in which limit one recovers exactly the flat space behavior: the translational invariance over the x^I directions is reinstated and the overall amplitude vanishes.

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A. Closed string mode expansions

Given the lightcone gauge fixed action (3.6) the closed string mode expansions are given by

$$x^I(\sigma,\tau) = \cos(m\tau)x_0^I + m^{-1}\sin(m\tau)p_0^I + i\sum_{n\neq 0}\frac{1}{\sqrt{2}}\omega_n^{-1}(\alpha_n^{1I}\tilde{\phi}_n + \alpha_n^{2I}\phi_n)\,; \tag{A.1}$$

$$\sqrt{2p^+}\theta^1(\sigma,\tau) = \theta_0^1\cos(m\tau) + \Pi\theta_0^2\sin(m\tau) + \sum_{n\neq 0} c_n\left(id_n\Pi\theta_n^2\phi_n + \theta_n^1\tilde{\phi}_n\right); \tag{A.2}$$

$$\sqrt{2p^+}\theta^2(\sigma,\tau) = \theta_0^2\cos(m\tau) - \Pi\theta_0^1\sin(m\tau) + \sum_{n\neq 0} c_n\left(-id_n\Pi\theta_n^1\tilde{\phi}_n + \theta_n^2\phi_n\right), \tag{A.3}$$

where the expansion functions are

$$\phi_n(\tau,\sigma) = e^{-i(w_n\tau+n\sigma)}\,, \qquad \tilde{\phi}_n(\tau,\sigma) = e^{-i(w_n\tau-n\sigma)}\,. \tag{A.4}$$

After canonical quantization we get the following (anti)commutators

$$\begin{aligned} \left[p_0^I, x_0^J\right] &= -i\delta^{IJ}\,, & \left[\alpha_m^{\mathcal{I}I}, \alpha_n^{\mathcal{J}J}\right] &= \omega_m\delta_{n+m,0}\delta^{\mathcal{I}\mathcal{J}}\delta^{IJ}\,, \\ \left\{\theta_0^{\mathcal{I}a}, \theta_0^{\mathcal{J}b}\right\} &= \delta^{\mathcal{I}\mathcal{J}}\delta^{ab}\,, & \left\{\theta_m^{\mathcal{I}a}, \theta_n^{\mathcal{J}b}\right\} &= \delta^{\mathcal{I}\mathcal{J}}\delta_{m+n,0}\delta^{ab}\,, \end{aligned} \tag{A.5}$$

where $\mathcal{I} = 1,2$. It is convenient to introduce creation and annihilation operators

$$a_0^I = \frac{1}{\sqrt{2m}}(p_0^I + imx_0^I), \qquad \bar{a}_0^I = \frac{1}{\sqrt{2m}}(p_0^I - imx_0^I), \qquad [\bar{a}_0^I, a_0^J] = \delta^{IJ}. \tag{A.6}$$

B. Evaluation of Dirichlet zero mode amplitude

In this appendix we discuss the evaluation of the Dirichlet zero mode part of the amplitude:

$$\mathcal{A}_D = \langle 0|e^{\frac{1}{2}(\bar{a}_0 + i\sqrt{2y_1})^2} e^{\ln(z)a_0\bar{a}_0} e^{\frac{1}{2}(a_0 - i\sqrt{2y_2})^2}|0\rangle\,, \tag{B.1}$$

where $y_i^{r'} = m(x_i^{r'})^2, i = 1,2$. To evaluate this we make use of the Campbell-Baker-Hausdorff formulae

$$e^P e^Q = e^{P + \mathcal{L}_{\frac{1}{2}P}(Q + \coth(\mathcal{L}_{\frac{1}{2}P})Q) + \cdots} = e^{Q + \mathcal{L}_{\frac{1}{2}Q}(-P + \coth(\mathcal{L}_{\frac{1}{2}Q})P) + \cdots}\,, \tag{B.2}$$

which contain all terms linear in the operator Q (P) in the first (second) formula. The Lie derivative is defined as

$$\mathcal{L}_{\frac{1}{2}P}Q = \frac{1}{2}[P,Q]\,, \tag{B.3}$$

and the hyperbolic cotangent should be evaluated as a power series expansion in

$$\mathcal{L}^n_{\frac{1}{2}P}Q = \left[\frac{1}{2}P, \left[\frac{1}{2}P, \ldots \left[\frac{1}{2}P, Q\right]\right]\right], \tag{B.4}$$

with n factors $\frac{1}{2}P$.[6] The formulae (B.2) are exact provided that all commutators involving more than one Q (P) vanish. Using these formulae one can show that

$$e^{\ln(z)a_0\bar{a}_0} e^{\frac{1}{2}(a_0 - i\sqrt{2y_2})^2}|0\rangle = e^{\frac{1}{2}(za_0 - i\sqrt{2y_2})^2} e^{\ln(z)a_0\bar{a}_0}|0\rangle = e^{\frac{1}{2}(za_0 - i\sqrt{2y_2})^2}|0\rangle\,, \tag{B.5}$$

[6] Recall that the expansion of the hyperbolic cotangent is $x\coth(x) = \sum_{n=0}^{\infty}\frac{2^{2n}}{(2n)!}B_{2n}x^{2n}$ where B_{2n} are Bernoulli numbers.

since $\bar{a}_0 |0\rangle = 0$. Thus (B.1) reduces to

$$\mathcal{A}_D(z, y_1, y_2) = \langle 0| e^{\frac{1}{2}(\bar{a}_0 + i\sqrt{2y_1})^2} e^{\frac{1}{2}(z a_0 - i\sqrt{2y_2})^2} |0\rangle. \tag{B.6}$$

The easiest way to compute this amplitude is to show that it satisfies a set of differential equations which can then be integrated. For instance, setting $y_1 = y_2 = 0$ and differentiating w.r.t. z yields

$$\partial_z \mathcal{A}_D(y_1{=}y_2{=}0) = zB \tag{B.7}$$

where $B = \langle 0| e^{\frac{1}{2}\bar{a}_0^2} a_0^2 e^{\frac{1}{2}z^2 a_0^2} |0\rangle$. Commuting a_0^2 to the left where it annihilates $\langle 0|$ yields $B = \mathcal{A}_D(y_1{=}y_2{=}0) + z^2 B$ and we finally arrive at

$$\partial_z \mathcal{A}_D(y_1{=}y_2{=}0) = \frac{z}{1-z^2} \mathcal{A}_D(y_1{=}y_2{=}0)\,. \tag{B.8}$$

Similar manipulations yield the following differential equations

$$\partial_{y_1} \mathcal{A}_D = \frac{1}{1-z^2}\left(z\sqrt{\frac{y_2}{y_1}} - 1\right)\mathcal{A}_D\,;\,, \tag{B.9}$$

$$\partial_{y_2} \mathcal{A}_D = \frac{1}{1-z^2}\left(z\sqrt{\frac{y_1}{y_2}} - 1\right)\mathcal{A}_D\,. \tag{B.10}$$

These equations together with $\mathcal{A}_D(z{=}y_1{=}y_2{=}0) = 1$ imply

$$\mathcal{A}_D = \frac{1}{(1-z^2)^{\frac{1}{2}}} e^{-\frac{1}{1-z^2}(y_1 + y_2 - 2z\sqrt{y_1 y_2})}\,. \tag{B.11}$$

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